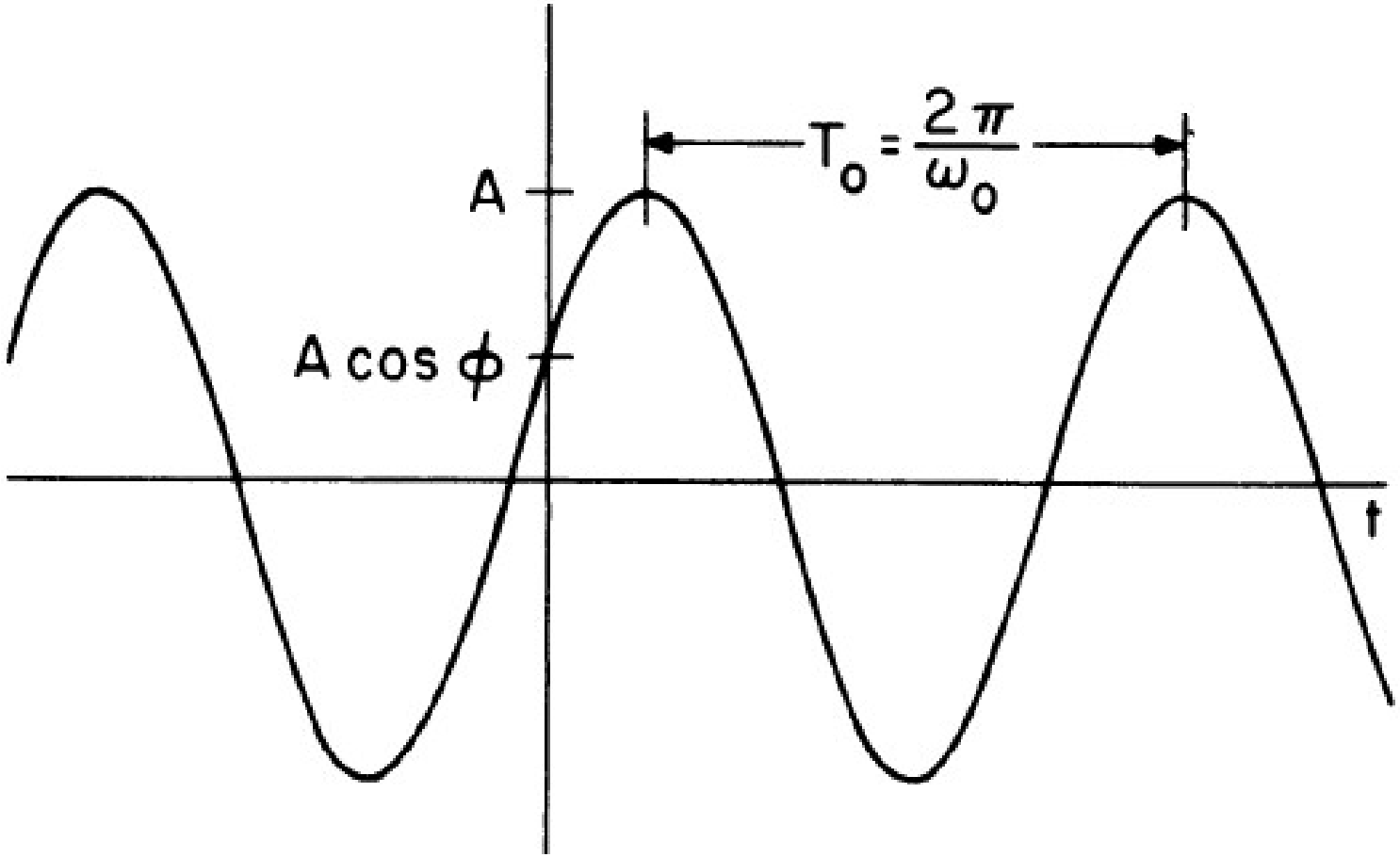
EE 362: Signals and Systems

Fundamental Concepts

Lecture 2

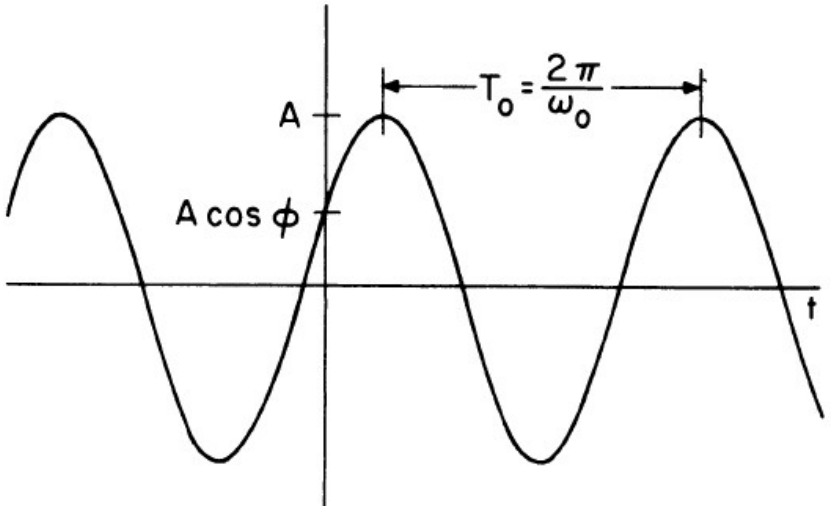
Additional reference: Signals and Systems, Professor Alan V. Oppenheim
http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/

Continuous-Time Sinusoidal Signals



$$x(t) = A\cos(\omega_o t + \phi)$$

Periodic Functions



$$x(t) = x(t + T_o) \qquad \text{period} \triangleq \text{smallest } T_o$$

$$A\cos[\omega_o t + \phi] = A\cos[\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi]$$

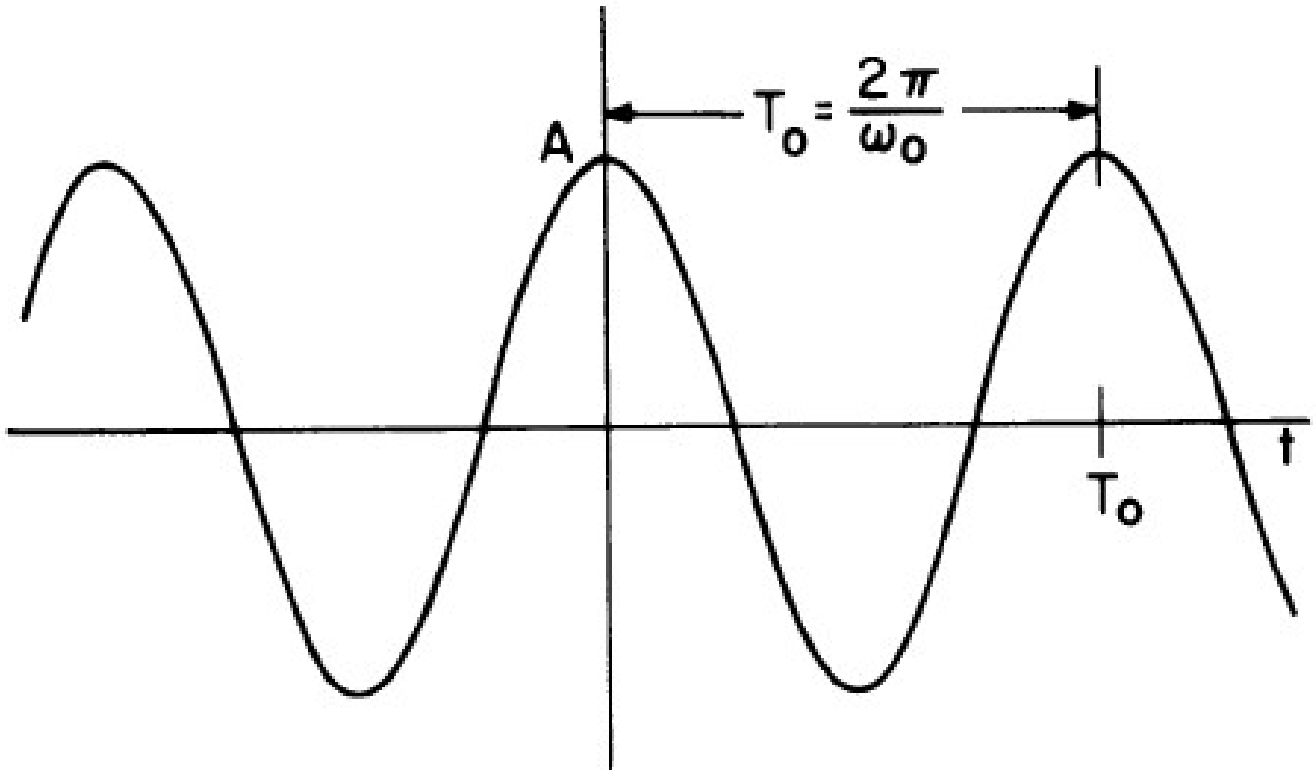
$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

Time Shift <=> Phase Change

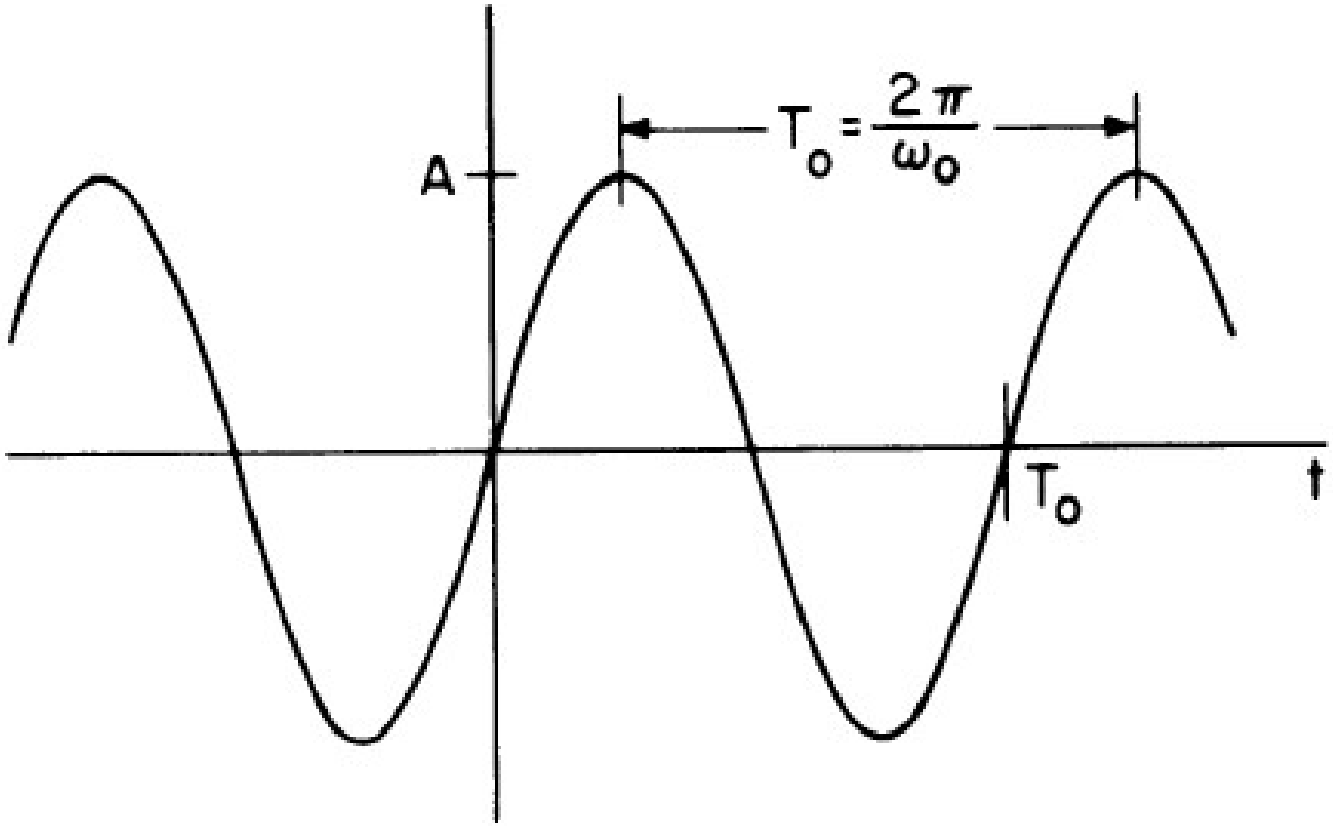
$$A\cos[\omega_o(t + t_o)] = A\cos[\omega_o t + \omega_o t_o]$$

$$A\cos[\omega_o(t + t_o) + \phi] = A\cos[\omega_o t + \omega_o t_o + \phi]$$

Even & Odd Functions



Even & Odd Functions

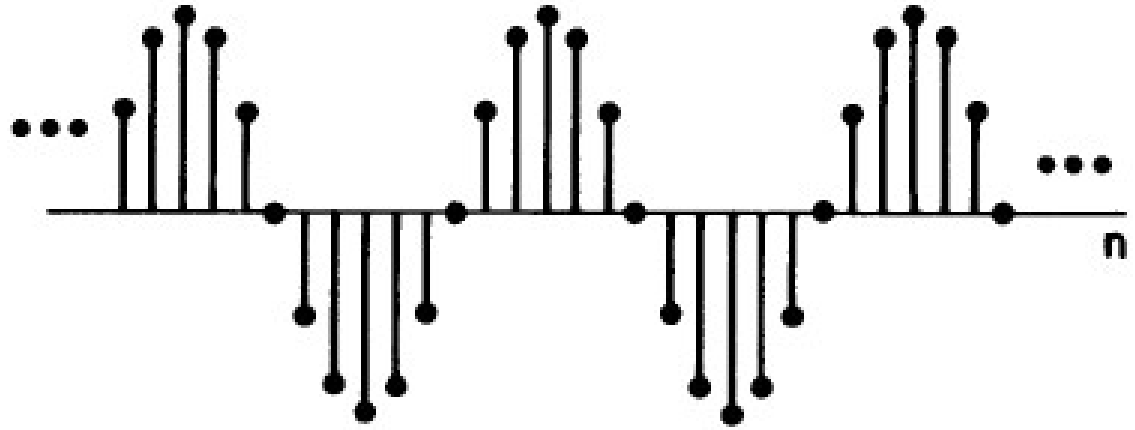


Discrete-Time Sinusoidal Signals

$$x[n] = A \cos(\Omega_0 n + \phi)$$

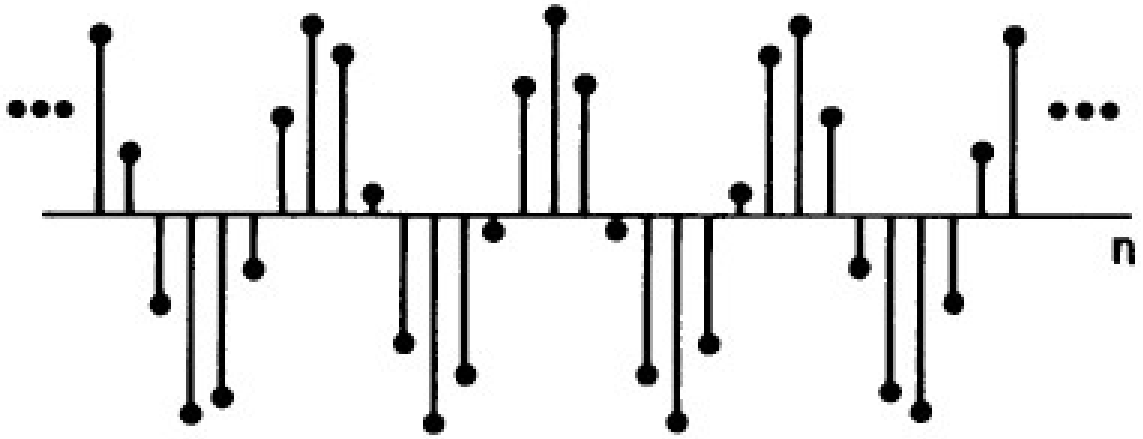
$\Omega_0 = \frac{2\pi}{12}$

$\phi = 0$



$\Omega_0 = \frac{8\pi}{31}$

$\phi = 0$



Periodicity

$$x[n] = A\cos(\Omega_o n + \phi)$$

Periodic?

$x[n] = x[n+N]$ smallest integer $N \triangleq$ period

$$A\cos[\Omega_o(n+N)+\phi] = A\cos[\Omega_o n + \underbrace{\Omega_o N} + \phi]$$

integer multiple of 2π ?

Periodic $=> \Omega_o N = 2\pi m$

$$N = \frac{2\pi m}{\Omega_o}$$

N,m must be integers

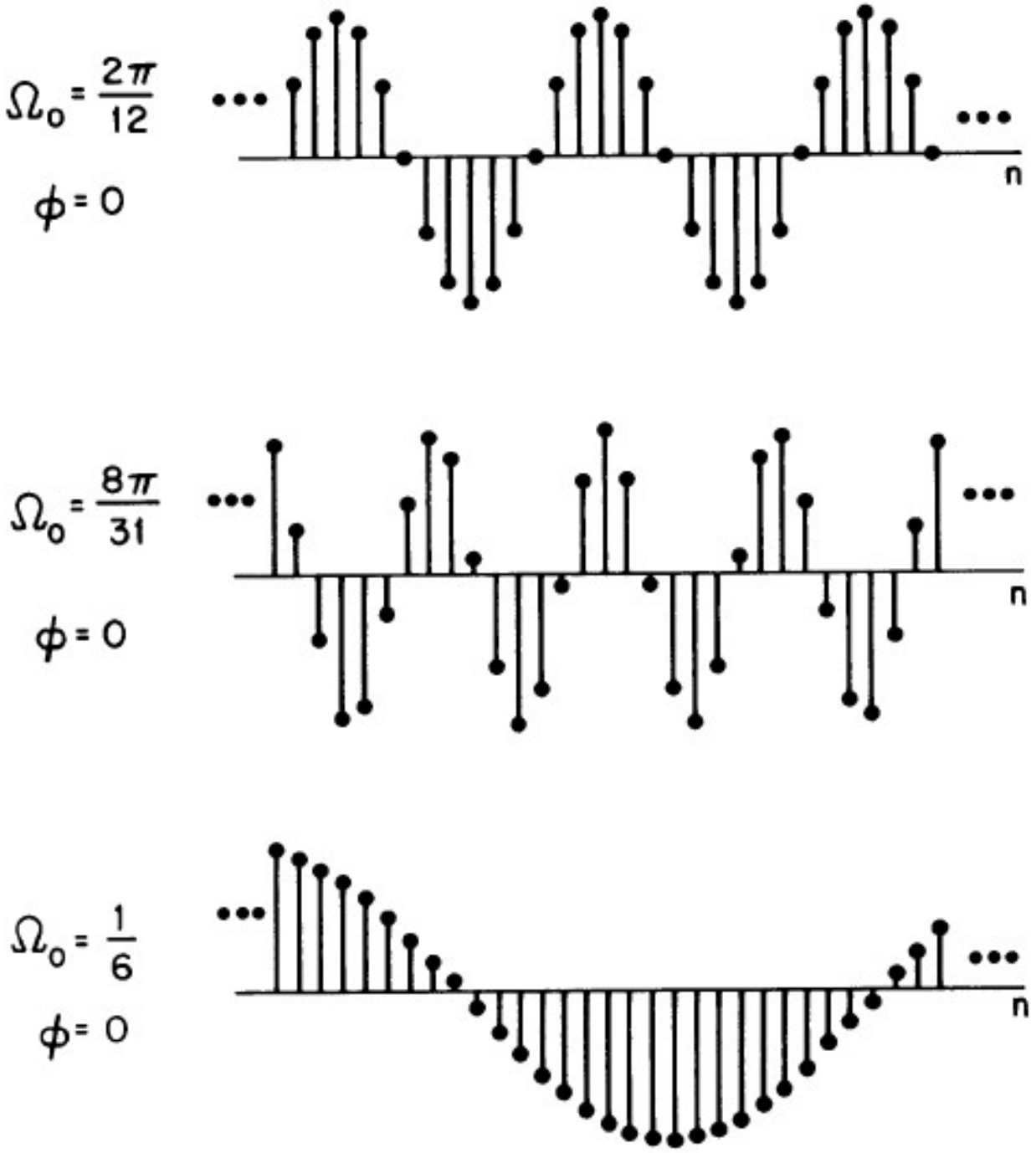
smallest N (if any) = period

Time Shift $\Rightarrow$ Phase Change

$$A \cos [\Omega_o(n + n_o)] = A \cos [\Omega_o n + \Omega_o n_o]$$

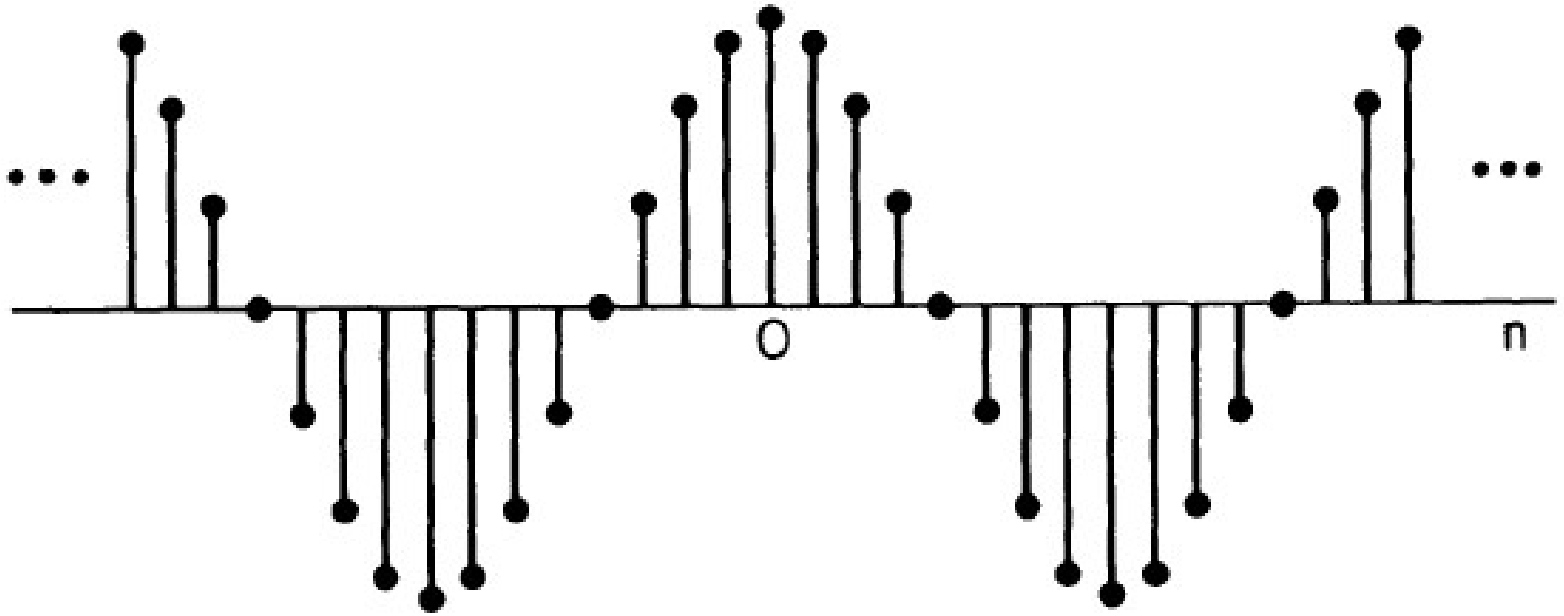
Time Shift $\overset{?}{\Leftarrow}$ Phase Change

$$A \cos [\Omega_o(n + n_o)] \overset{?}{=} A \cos [\Omega_o n + \phi]$$


$\Omega_0 = \frac{2\pi}{12}$
$\phi = 0$
n
$\Omega_0 = \frac{8\pi}{31}$
$\phi = 0$
n
$\Omega_0 = \frac{1}{6}$
$\phi = 0$
n

Even & Odd Functions

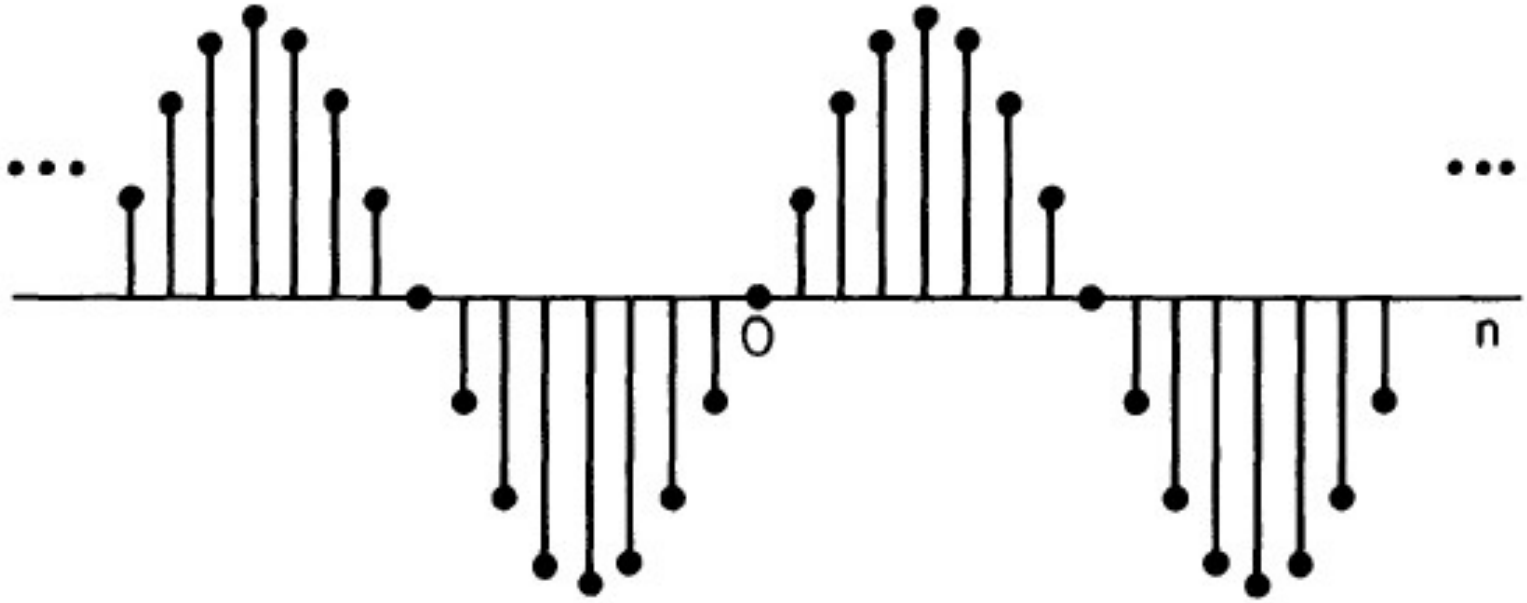
$$x[n] = A \cos \Omega_0 n$$



$$\Omega_0 = \frac{\pi}{8}$$

$$x[n] = \begin{cases} A\cos\left(\Omega_0 n - \frac{\pi}{2}\right) \\ A\sin \Omega_0 n \\ A\cos\left[\Omega_0 (n - n_0)\right] \end{cases}$$

$n_0 = ?$



$\Omega_0 = \frac{\pi}{8}$

Continuous time:

$x_1(t) = A\cos(\omega_1 t + \phi)$ If $\omega_2 \neq \omega_1$

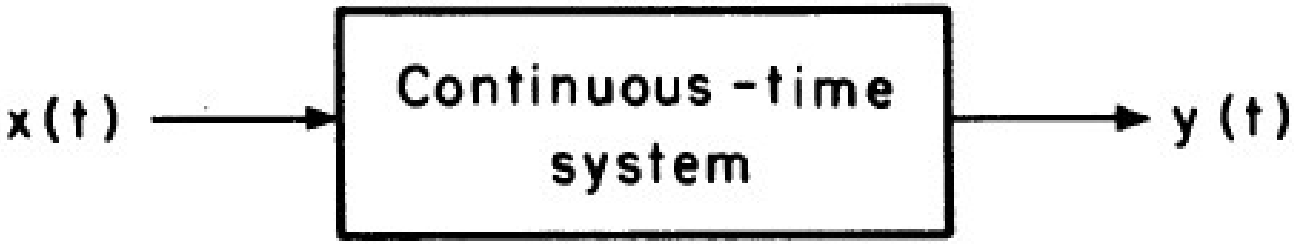
$x_2(t) = A\cos(\omega_2 t + \phi)$ Then $x_2(t) \neq x_1(t)$

Discrete time:

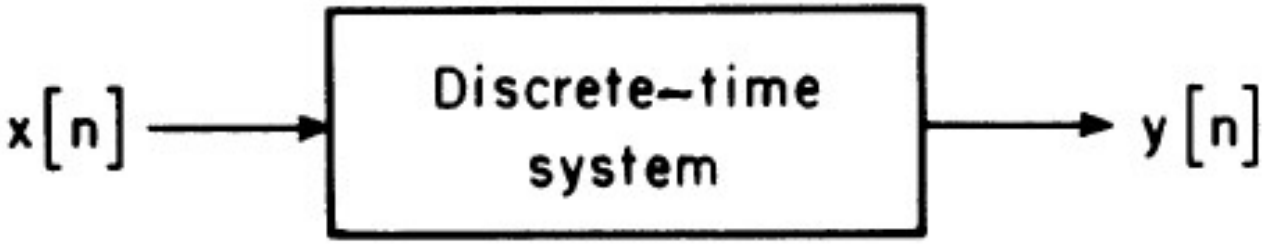
$x_1[n] = A\cos[\Omega_1 n + \phi]$ If $\Omega_2 = \Omega_1 + 2\pi m$

$x_2[n] = A\cos[\Omega_2 n + \phi]$ Then $x_2[n] = x_1[n]$

Systems

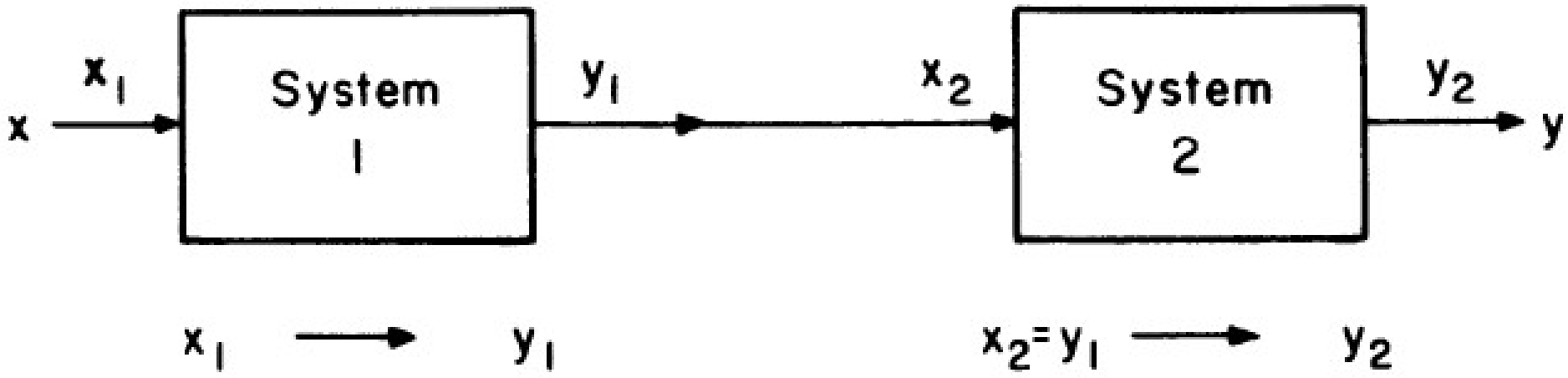


$x(t) \longrightarrow y(t)$

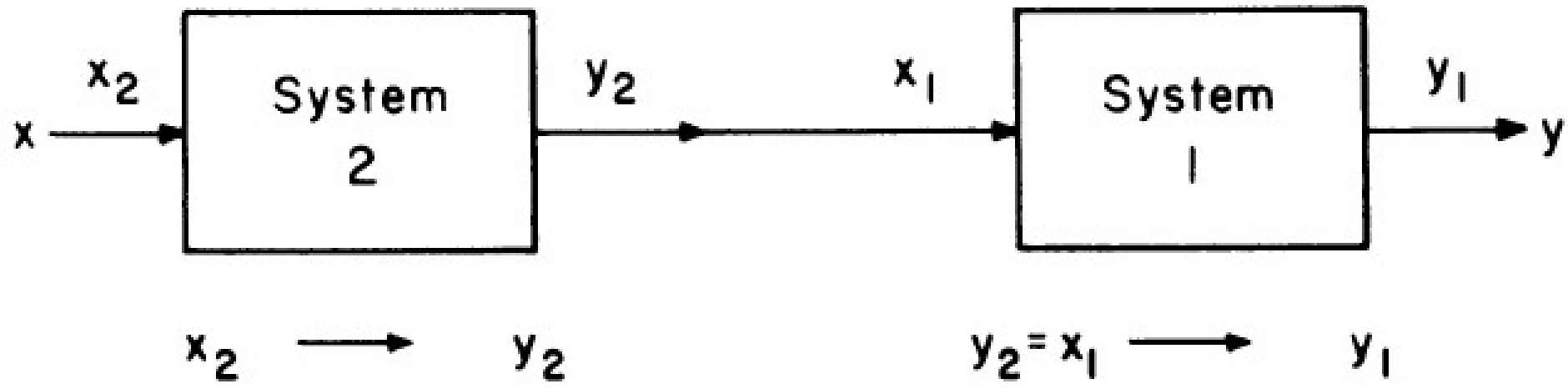


$x[n] \longrightarrow y[n]$

Cascade

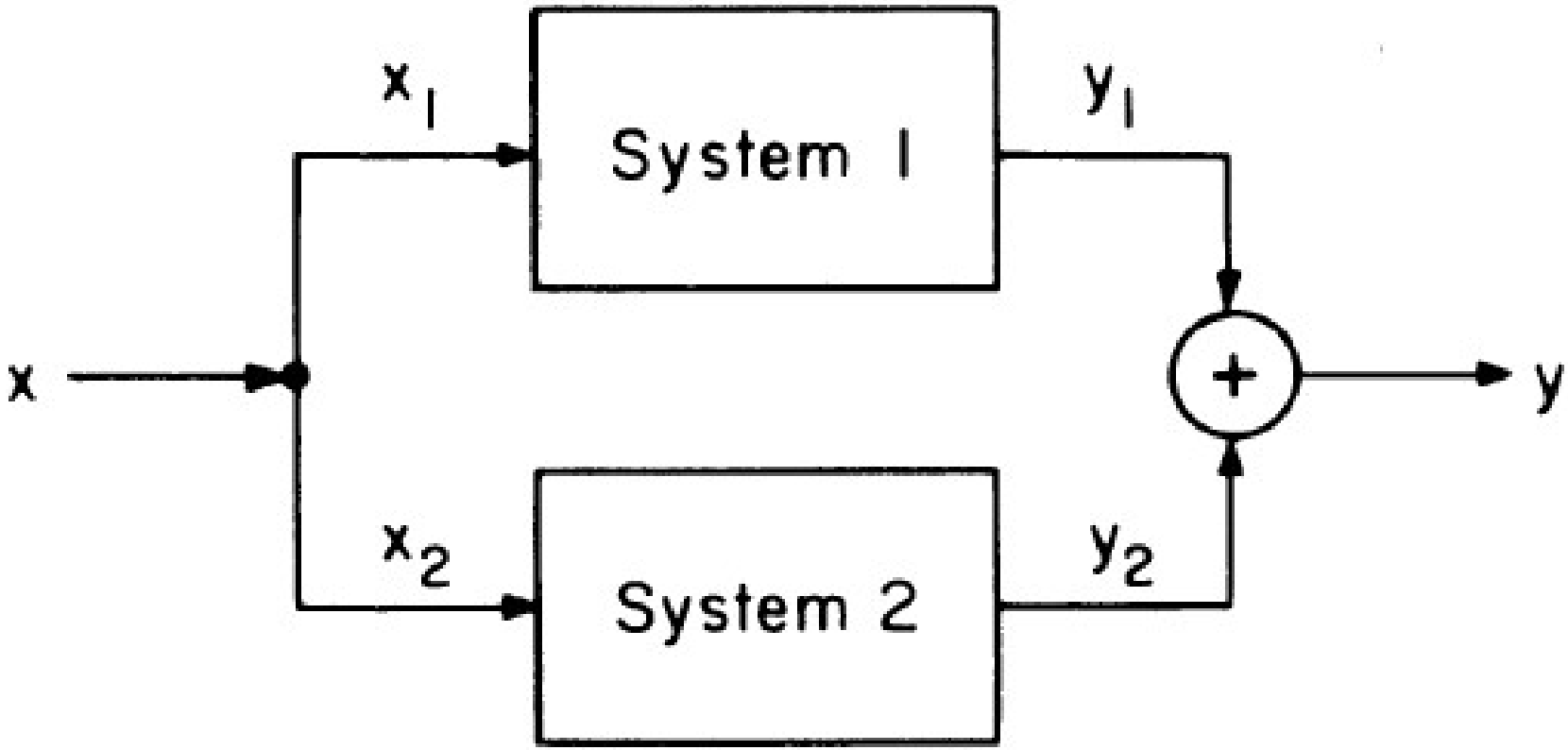


$x_1 \longrightarrow y_1$ $\qquad$ $x_2 = y_1 \longrightarrow y_2$



$x_2 \longrightarrow y_2$ $\qquad$ $y_2 = x_1 \longrightarrow y_1$

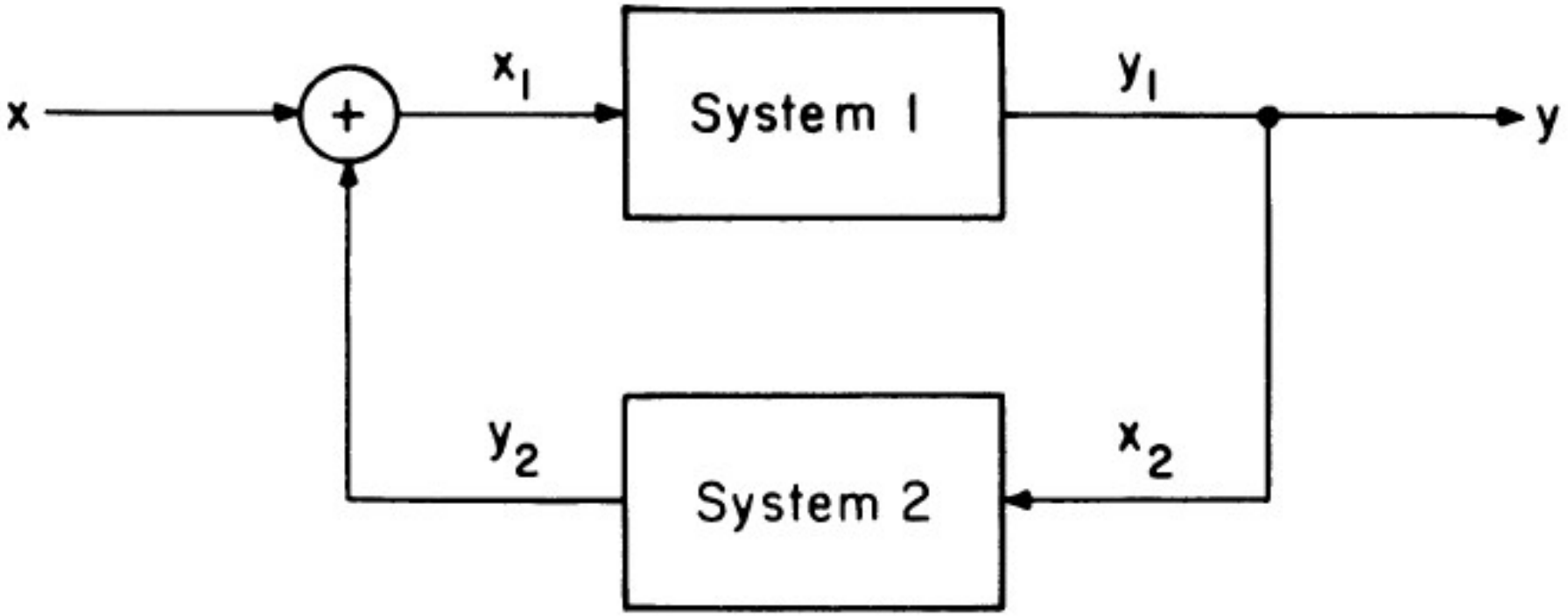
parallel



$x_1 = x_2 = x$

$y = y_1 + y_2$

feedback



$x_1 = x + y_2$

$y = y_1$

$x_2 = y_1$

System Properties

- Memoryless
- Linearity
- Causality
- Time invariance

- Memoryless

Examples: Are the following systems memoryless?

$y(t) = x^2(t)$

$y[n] = x^2[n]$

$y[n] = x[n-1]$

$y(t) = \int_{-\infty}^{t} x^2(t)dt$

- Linearity

$x_1(t) \rightarrow y_1(t)$ & $x_2(t) \rightarrow y_2(t)$

$ax_1(t) + b\,x_2(t) \rightarrow ay_1(t) + b\,y_2(t)$

Examples: Are the following systems linear?

$y[n] = x^2[n]$

$y(t) = \int_{-\infty}^{t} x(\tau)d\,\tau$

$y[n] = 2\,x[n] + 4$

- Causality

Output at any time depends only on input prior or equal to that time.

Examples: Are the following systems causal?

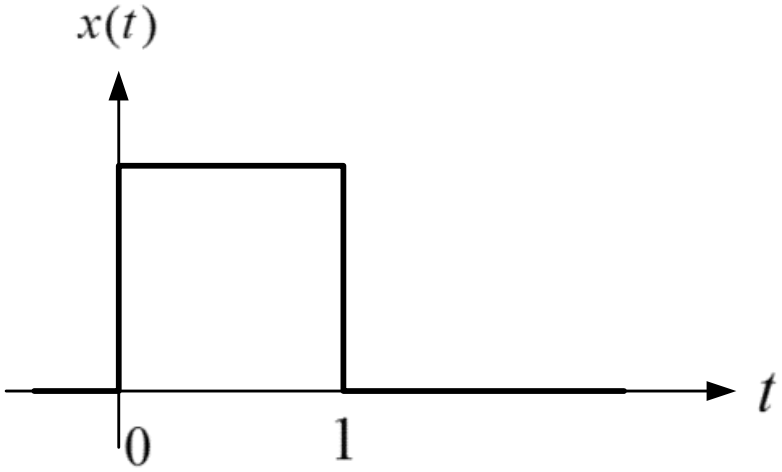
$y[n] = 1/3\{x(n-1)+x(n)+x(n+1)\}$

$y[n] = 1/3\{x(n-2)+x(n-1)+x(n)\}$

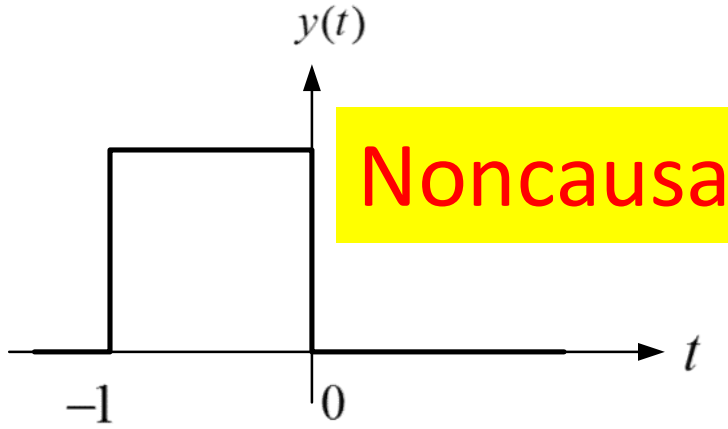
$y[n] = 1/3\{x(n)+x(n+1)+x(n+2)\}$

Causal (nonanticipatory) Systems

Example 1: $y(t) = x(t+1)$

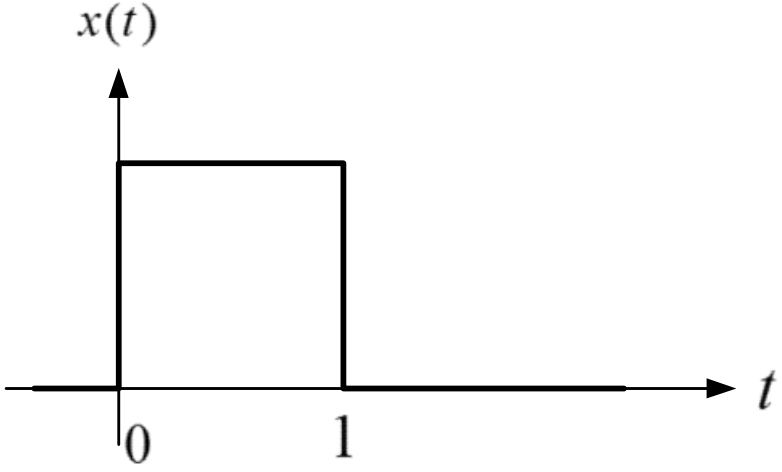


Ideal predictor

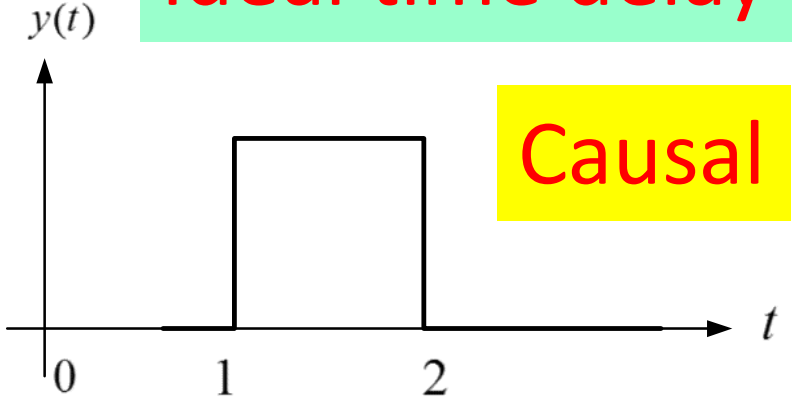


Noncausal

Example 2: $y(t) = x(t-1)$



Ideal time delay



Causal

- Time Invariance

A system is said to be time invariant if for any input x(t) and any t_1, the shifted input x(t-t_1) is equal to y(t-t_1), where y(t) is the response to x(t).

Example: y(t) = t x(t); is it time invariant?

Time Invariant Systems

Example 1: $y(t) = tx(t)$

$$x(t - t_1) \quad \rightarrow \quad tx(t - t_1)$$

But $y(t - t_1) = (t - t_1)x(t - t_1)$ ∴ Time varying

Example 2: $y(t) = 2x(t)$

$$x(t - t_1) \quad \rightarrow \quad x(t - t_1)$$

And $y(t - t_1) = x(t - t_1)$

∴ Time invariant