EE 362: Signals and Systems

Introduction

Lecture 1

Additional reference: Signals and Systems, Professor Alan V. Oppenheim
http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/

Preliminaries

- Derivatives
- Integrals
- Complex numbers
- Sinusoidal functions

Signals

- Signals are functions of independent variables that carry some type of information

Examples of signal include:

Electrical signals

– Voltages and currents in a circuit

Acoustic signals

– Acoustic pressure (sound) over time

Mechanical signals

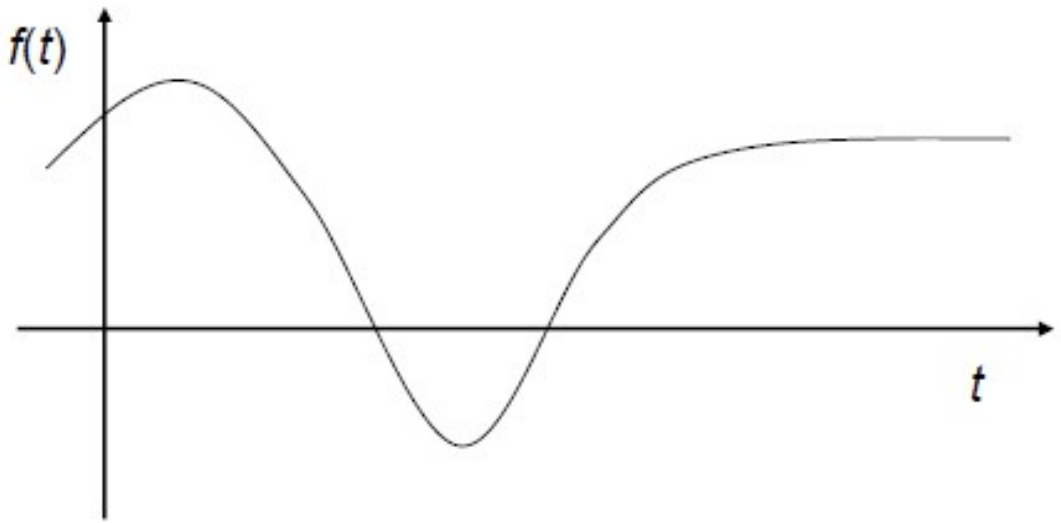
– Velocity of a car over time

Video signals

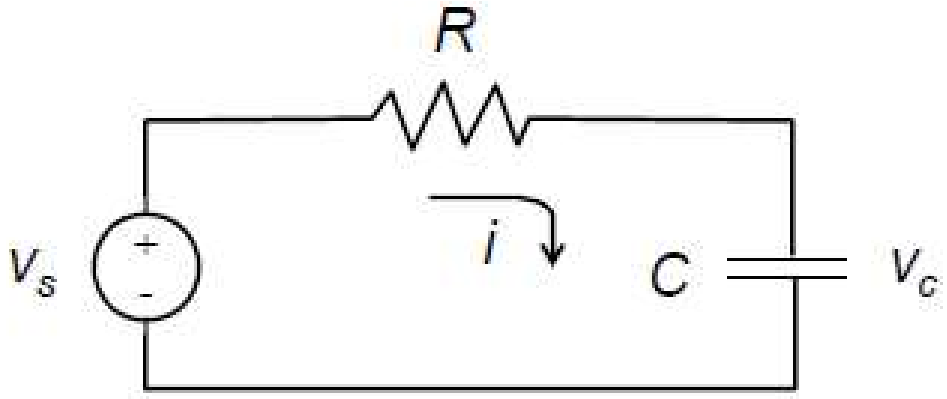
– Intensity level of a pixel (camera, video) over time

Representation of Signals

- Mathematically, signals are represented as a function of one or more **independent variables.**
- For instance a black & white video signal intensity is dependent on *x, y coordinates and time t: f(x,y,t)*
- On this course, we shall be exclusively concerned with signals that are a function of a single variable: time



Example: Signals in an Electrical Circuit

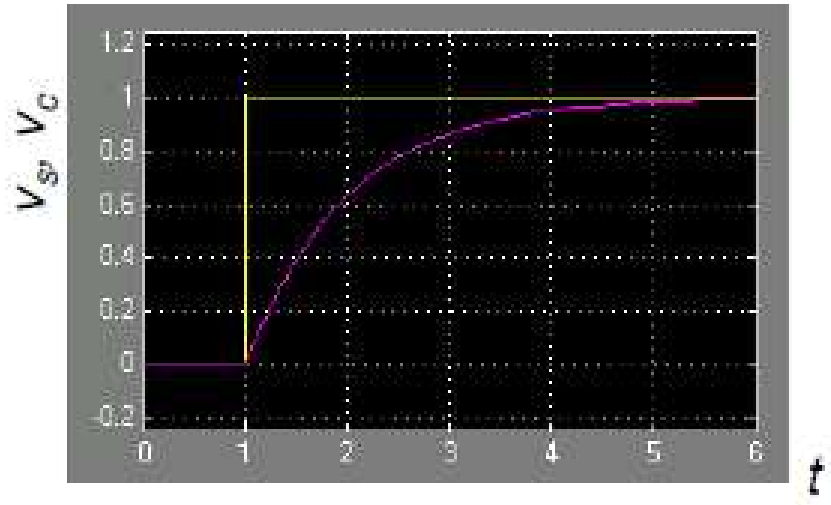


$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

The signals v_c and v_s are patterns of variation over time



- Step (signal) v_s at t=1
- RC = 1
- First order (exponential) response for v_c

Note, we could also have considered the voltage across the resistor or the current as signals

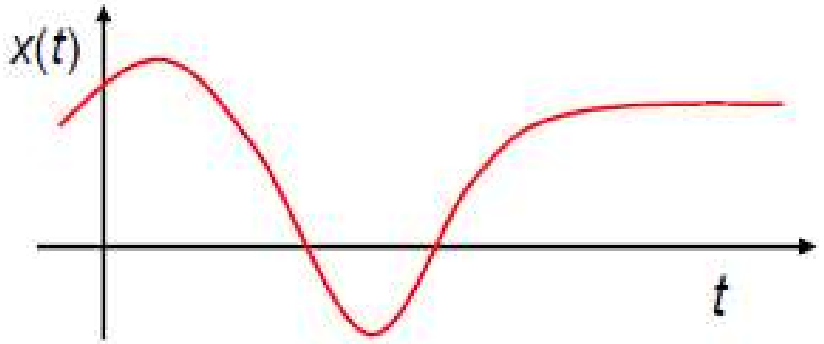
Continuous & Discrete-Time Signals

Continuous-Time Signals

Most signals in the real world are continuous time, as the scale is infinitesimally fine.

Eg voltage, velocity,

Denote by $x(t)$, where the time interval may be bounded (finite) or infinite

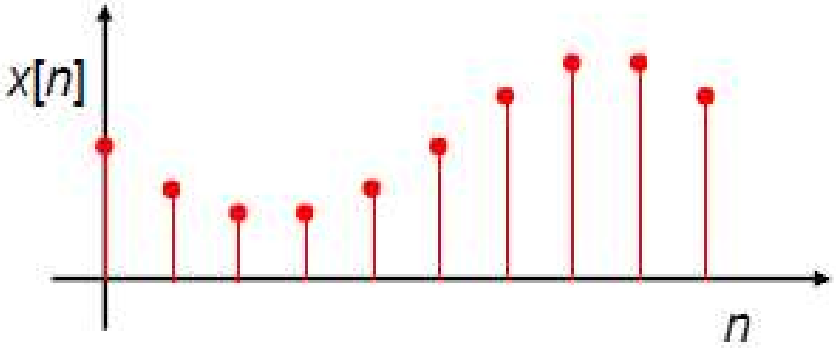


Discrete-Time Signals

Some real world and many digital signals are discrete time, as they are sampled

E.g. pixels, daily stock price (anything that a digital computer processes)

Denote by $x[n]$, where n is an integer value that varies discretely



Sampled continuous signal

$x[n] = x(nk)$ – k is sample time

Signal Properties

On this course, we shall be particularly interested in signals with certain properties:

Periodic signals: a signal is periodic if it repeats itself after a fixed period T, i.e. $x(t) = x(t+T)$ for all t. A $\sin(t)$ signal is periodic.

Even and odd signals: a signal is even if $x(-t) = x(t)$ (i.e. it can be reflected in the axis at zero). A signal is odd if $x(-t) = -x(t)$. Examples are $\cos(t)$ and $\sin(t)$ signals, respectively.

Exponential and sinusoidal signals: a signal is (real) exponential if it can be represented as $x(t) = Ce^{at}$. A signal is (complex) exponential if it can be represented in the same form but C and a are complex numbers.

Step and pulse signals: A pulse signal is one which is nearly completely zero, apart from a short spike, $\delta(t)$. A step signal is zero up to a certain time, and then a constant value after that time, $u(t)$.

These properties define a large class of tractable, useful signals and will be further considered in the coming lectures

What is a System?

- Systems process input signals to produce output signals

Examples:

- A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
- A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
- A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?

A system takes a signal as an input and transforms it into another signal



In a very broad sense, a system can be represented as the ratio of the output signal over the input signal

That way, when we "multiply" the system by the input signal, we get the output signal

This concept will be firmed up in the coming weeks

Properties of a System

On this course, we shall be particularly interested in signals with certain properties:

- **Causal**: a system is causal if the output at a time, only depends on input values up to that time.
- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs
- **Time-invariance**: a system is time invariant if the system's output is the same, given the same input signal, regardless of time.

These properties define a large class of tractable, useful systems and will be further considered in the coming lectures

How Are Signal & Systems Related (i)?

How to design a system to process a signal in particular ways?

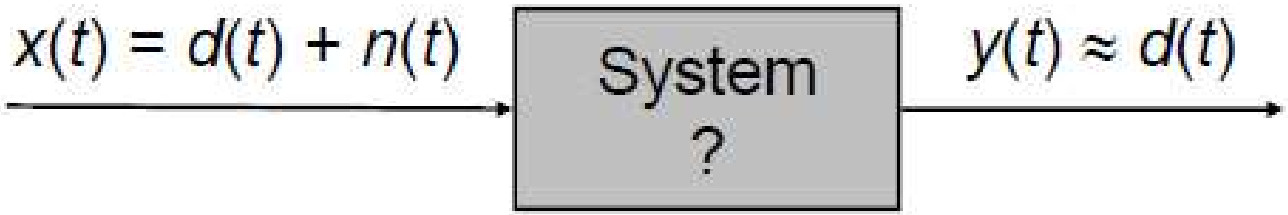
Design a system to restore or enhance a particular signal

- Remove **high frequency** background communication noise
- Enhance **noisy** images from spacecraft

Assume a signal is represented as

$x(t) = d(t) + n(t)$

Design a system to remove the unknown "noise" component $n(t)$, so that $y(t) \approx d(t)$



How Are Signal & Systems Related (ii)?

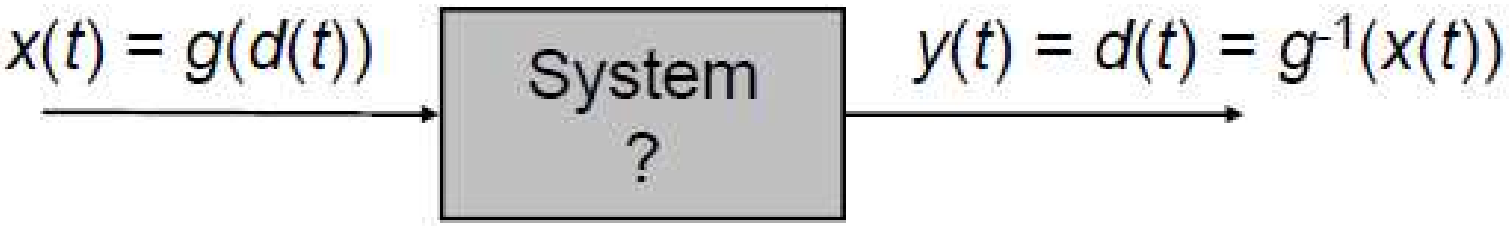
How to design a system to extract specific pieces of information from signals

- Estimate the heart rate from an electrocardiogram
- Estimate economic indicators (bear, bull) from stock market values

Assume a signal is represented as

$x(t) = g(d(t))$

Design a system to "invert" the transformation $g()$, so that $y(t) = d(t)$



How Are Signal & Systems Related (iii)?

How to design a (dynamic) system to modify or control the output of another (dynamic) system

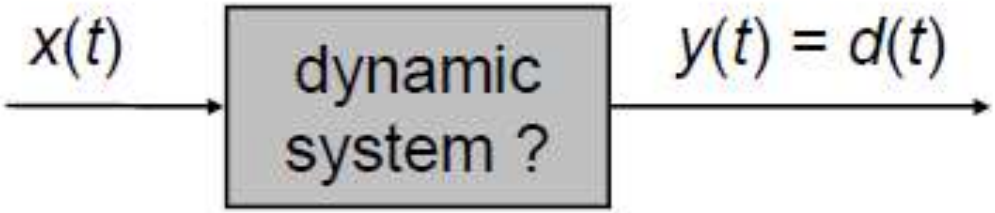
- Control an aircraft's altitude, velocity, heading by adjusting throttle, rudder, ailerons
- Control the temperature of a building by adjusting the heating/cooling energy flow.

Assume a signal is represented as

$x(t) = g(d(t))$

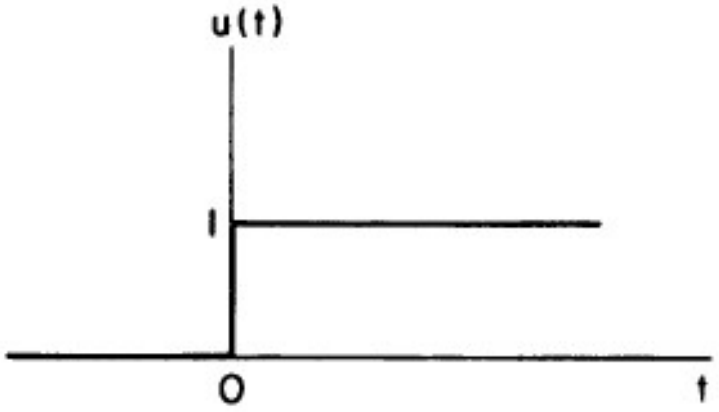
Design a system to "invert" the transformation $g()$, so that

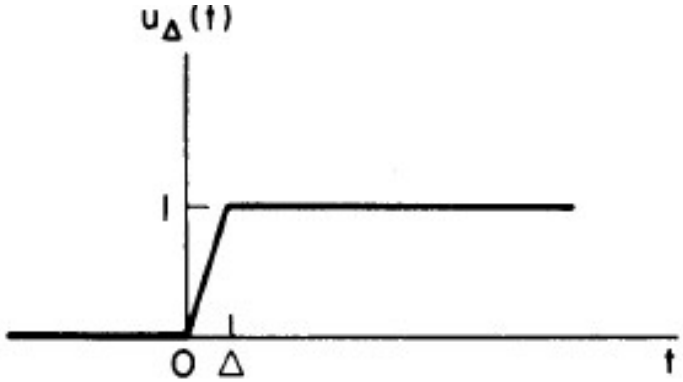
$y(t) = d(t)$



Continuous-Time Signals

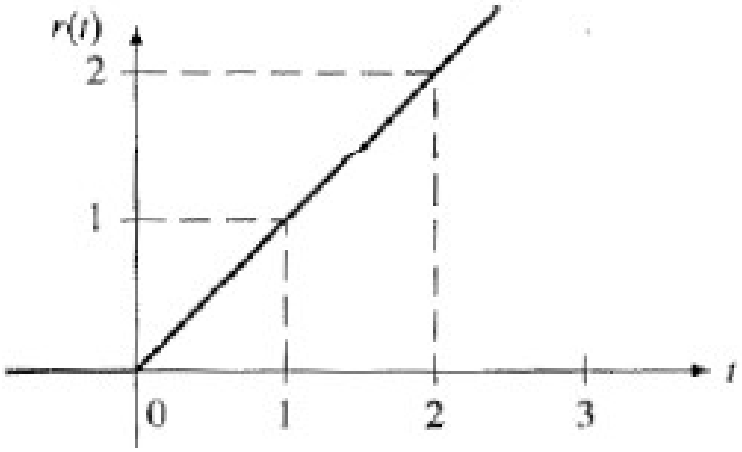
Unit Step Function u(t):



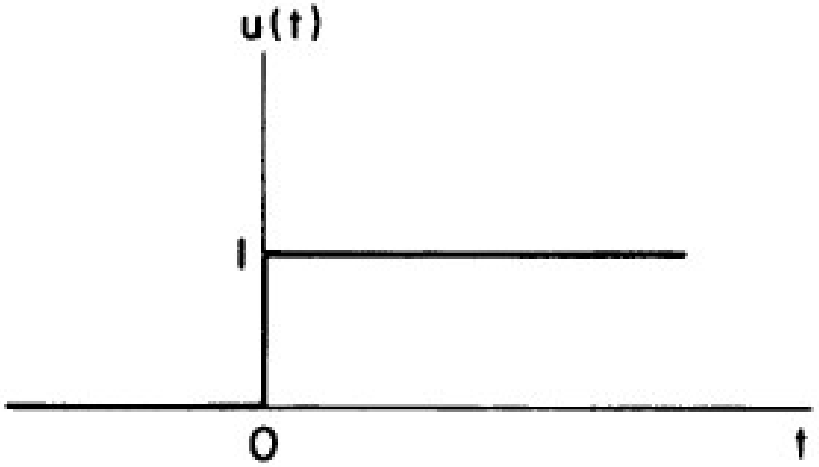


$u(t) = u_\Delta(t)$ as $\Delta \to 0$

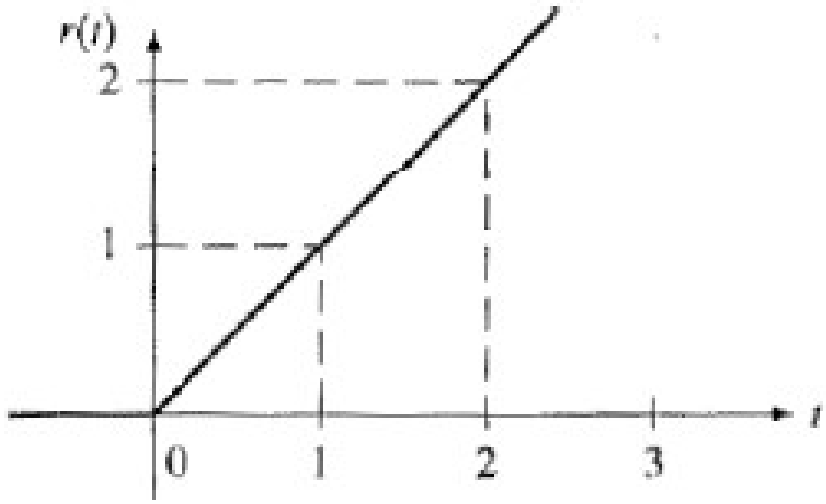
Unit Ramp Function r(t):

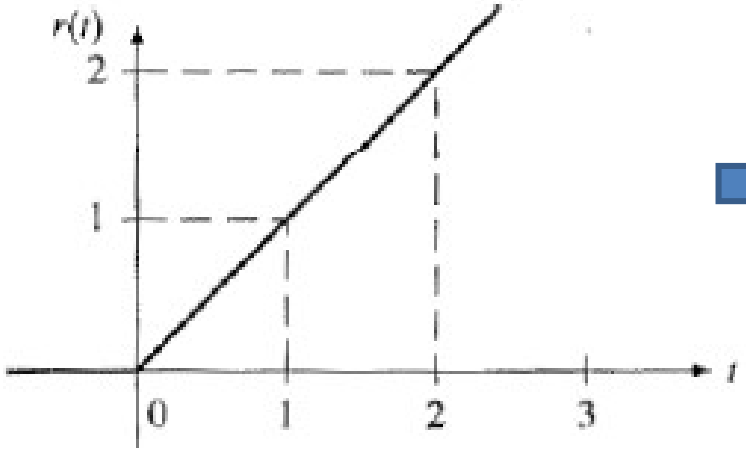


The relationship between the unit-step function *u(t) and the* unit-ramp function *r(t).*

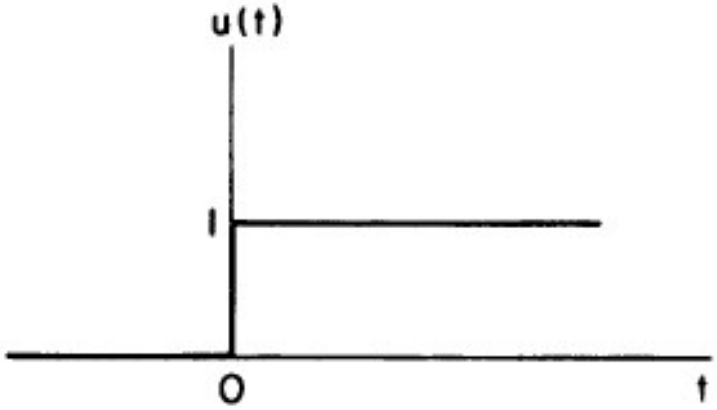


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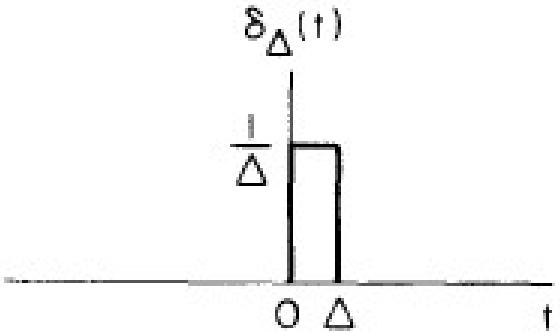


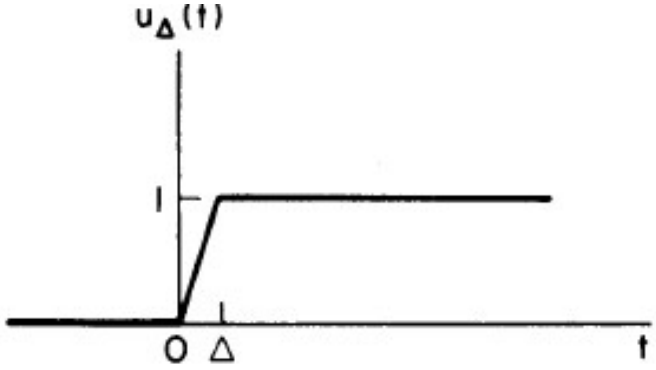


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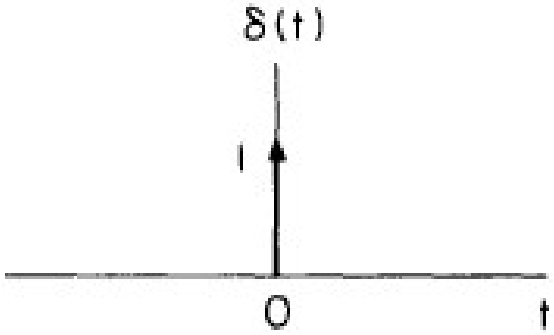


Unit Impulse Function δ(t):

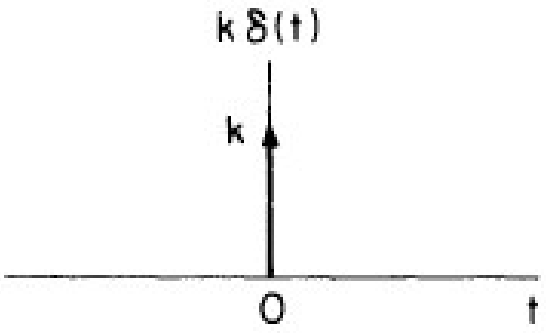




$u(t) = u_\Delta(t)$ as $\Delta \to 0$



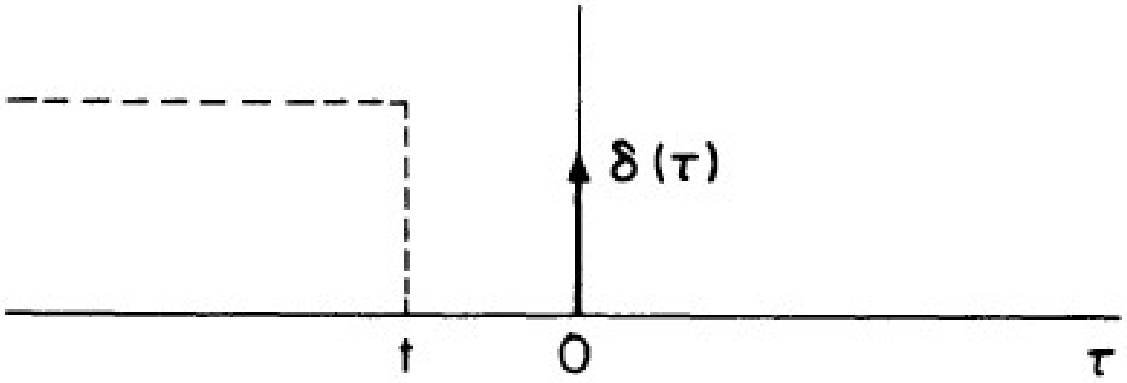
$$\delta(t) = \frac{du(t)}{dt}$$

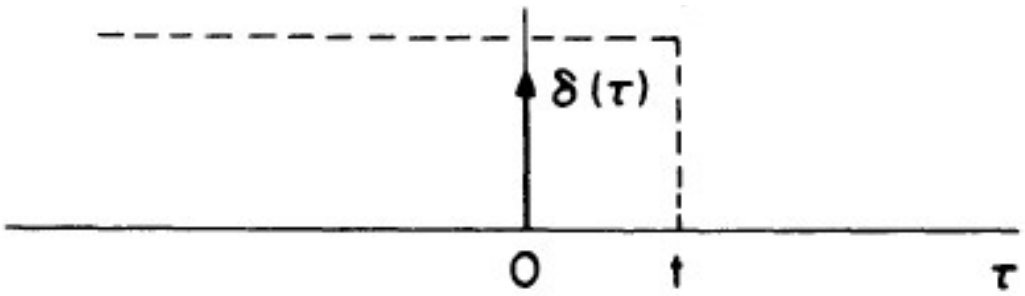


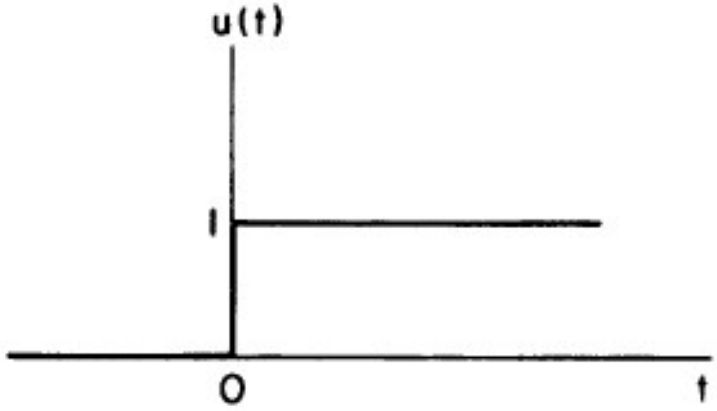
$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

$$\delta(t) = \delta_\Delta(t) \text{ as } \Delta \to 0$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$





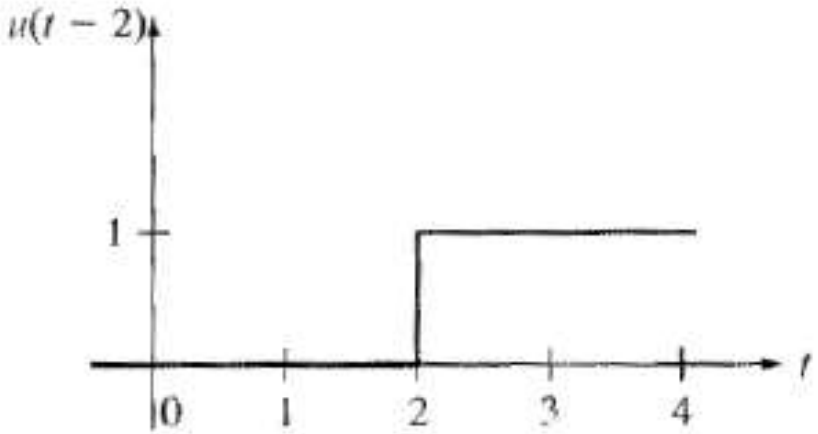


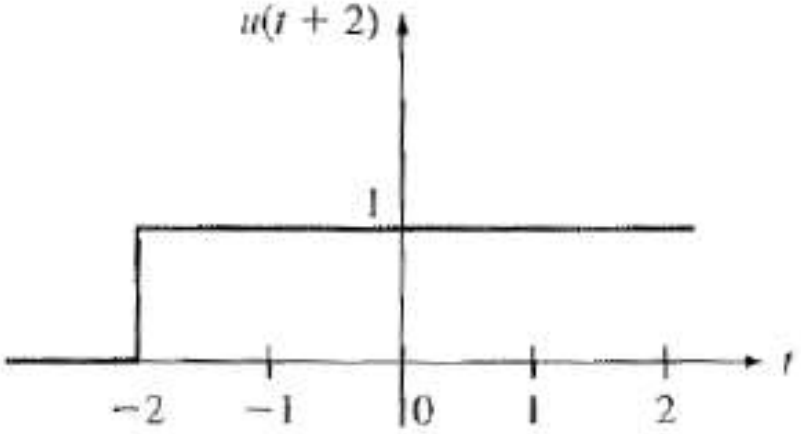
Time-shifted Signals: It's often necessary to consider a time-shifted version of $x(t)$

$x(t-t_1)$, $t_1 \geq 0$ → the signal $x(t-t_1)$ is $x(t)$ shifted to the right by t_1

$x(t+t_1)$, $t_1 \geq 0$ → the signal $x(t+t_1)$ is $x(t)$ shifted to the left by t_1

Example: Find $u(t-2)$ and $u(t+2)$? (Just consider the logic!)





Shifting Property of the impulse $\delta(t)$:

$$f(\lambda)\delta(\lambda - t_1) = f(t_1)\delta(\lambda - t_1)$$

$$\int_{t_1-\varepsilon}^{t_1+\varepsilon} f(\lambda)\delta(\lambda - t_1)d\lambda = f(t_1) \qquad \varepsilon > 0$$

Proof: $\delta(\lambda\text{-}t_1)$ is zero for $\lambda \neq t_1$, therefore:

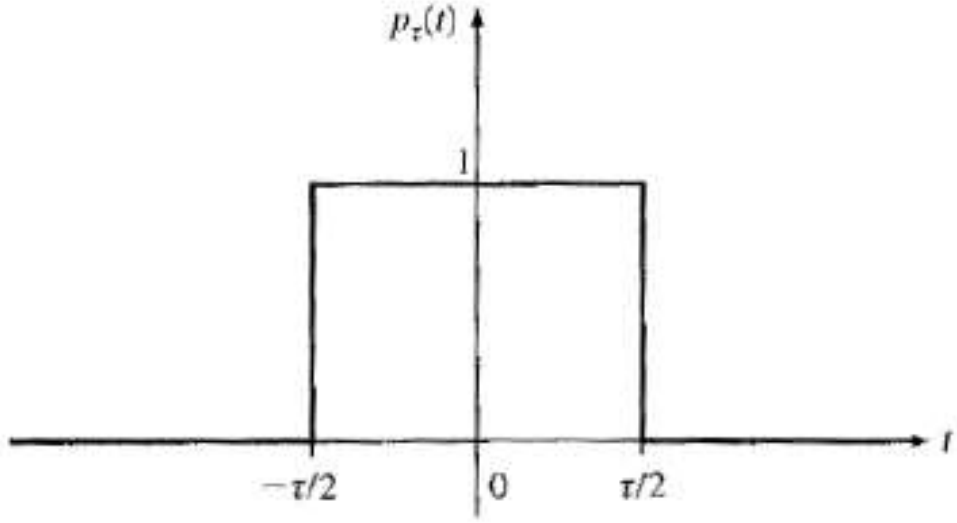
$$f(\lambda).\delta(\lambda\text{-}t_1) = f(t_1).\delta(\lambda\text{-}t_1)$$

$$\int_{t_1-\varepsilon}^{t_1+\varepsilon} f(\lambda)\delta(\lambda - t_1)d\lambda = f(t_1)\int_{t_1-\varepsilon}^{t_1+\varepsilon} \delta(\lambda - t_1)d\lambda = f(t_1)$$

Rectangular Pulse function:

$$P_\tau(t) = \begin{cases} 1 & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

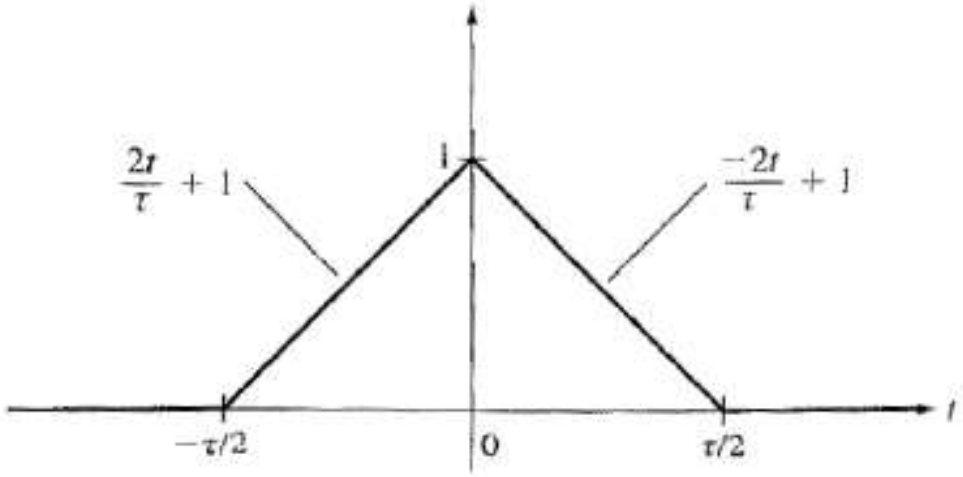
$$P_\tau(t) = u(t+\frac{\tau}{2}) - u(t-\frac{\tau}{2})$$



Triangular Pulse function:

$$f(t) = \begin{cases} 1+\frac{2t}{\tau} & -\frac{\tau}{2} \le t \le 0 \\ 1-\frac{2t}{\tau} & 0 \le t \le \frac{\tau}{2} \\ 0, & \text{others} \end{cases}$$

$$f(t) = (1-\frac{2|t|}{\tau})P_\tau(t)$$

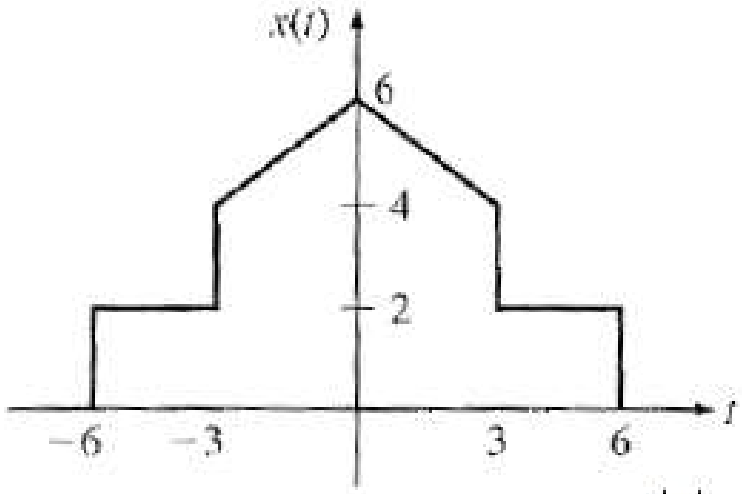


Example: x(t)is given below.

a) Sketch x(-t), x(-t+3), x(2t)

b) Find a combination of $P_\tau(t)$ and $(1-\frac{2|t|}{\tau})P_\tau(t)$ that gives x(t)

c) Plot this using MATLAB



$$f(t) = 2P_{12}(t) + 2P_6(t) + 2(1-\frac{2|t|}{6})P_6(t)$$

```
t = [-7 -6 -6 -3 -3 0 3 3 6 6 7];
x = [0  0  2 2 4 6 4 2 2 0 0];
plot(t,x)
```

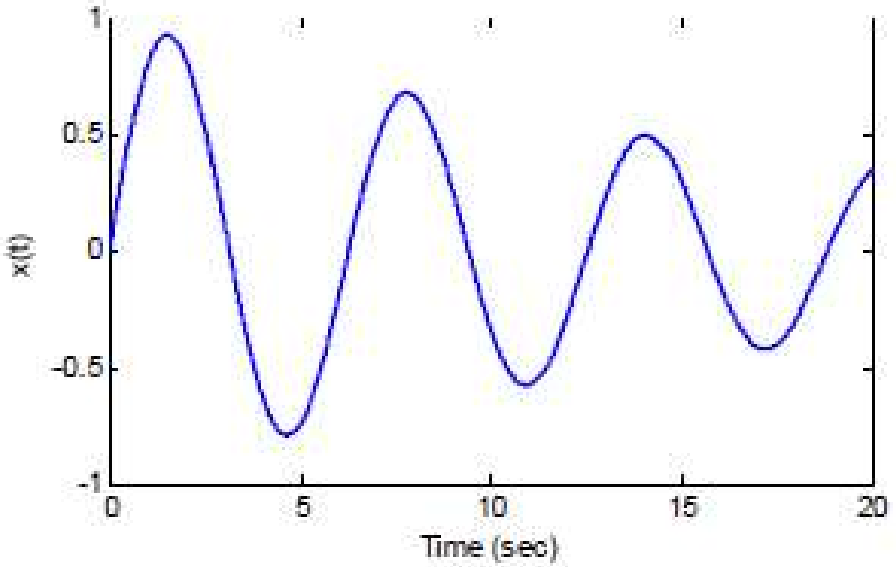
Using MATLAB:

Using MATLAB for plotting continuous-time signals

$$x(t) = e^{-0.05t}\sin(t)$$

MATLAB program:

```
t=0:0.01:30;
x=exp(-0.05*t).*sin(t);
plot(t,x)
axis([0 20 -1 1])
xlabel('Time (sec)')
ylabel('x(t)')
```



See Section 1.1.7 of the text book for more information.

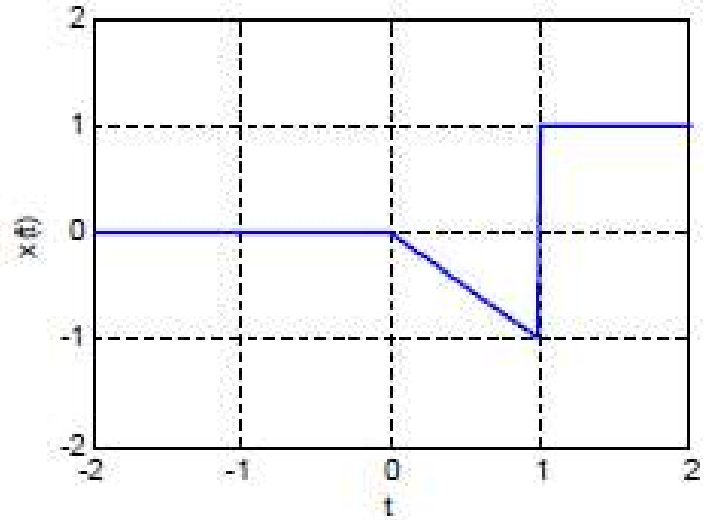
Example: Sketch the following continuous-time signal.

a) $x(t)=(t+1)u(t-1)-tu(t)$

Hint: $(t+1)u(t-1)=(t-1)\,u(t-1)+2\,u(t-1)$

MATLAB program:

```
a=0;
for t=-2:0.01:2
    a=a+1;
    x(a)=(t+1)*u(t-1)-t*u(t);
end
t=-2:0.01:2;
plot(t,x, 'LineWidth',2)
ylabel('x(t)')
xlabel('t')
```

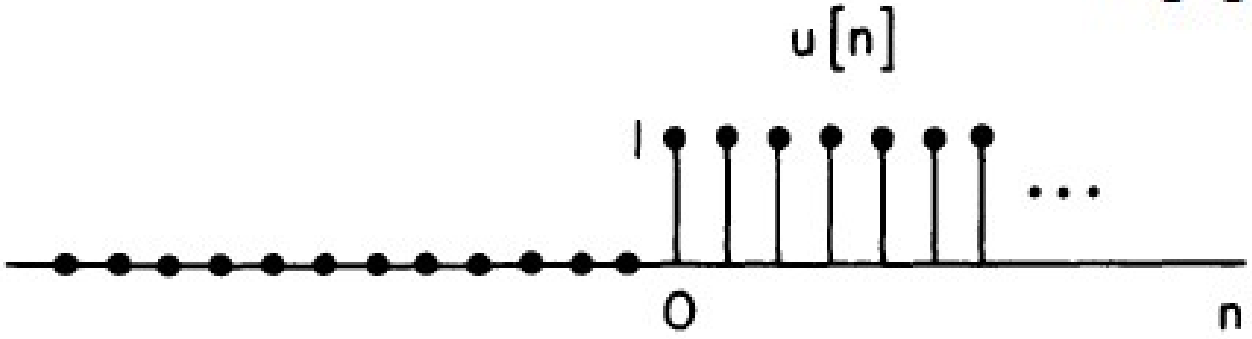


Discrete-Time Signals

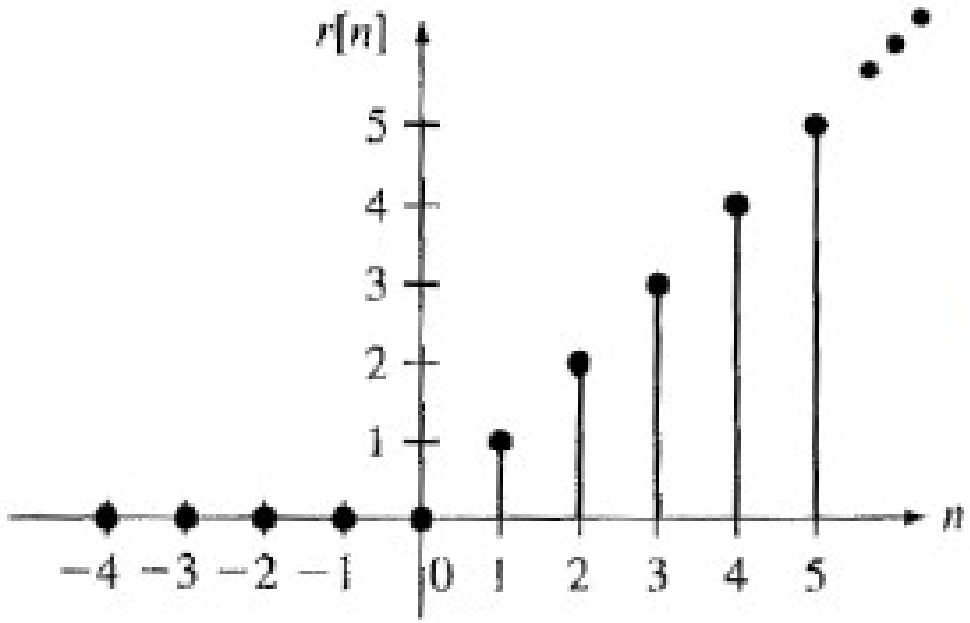
We use *x[n] to show discrete signals, where n is an integer* number

Unit Step Function u[n]:

$$u[n] = \begin{cases} 1, & n = 0, 1, \ldots \\ 0, & n = -1, -2, \ldots \end{cases}$$

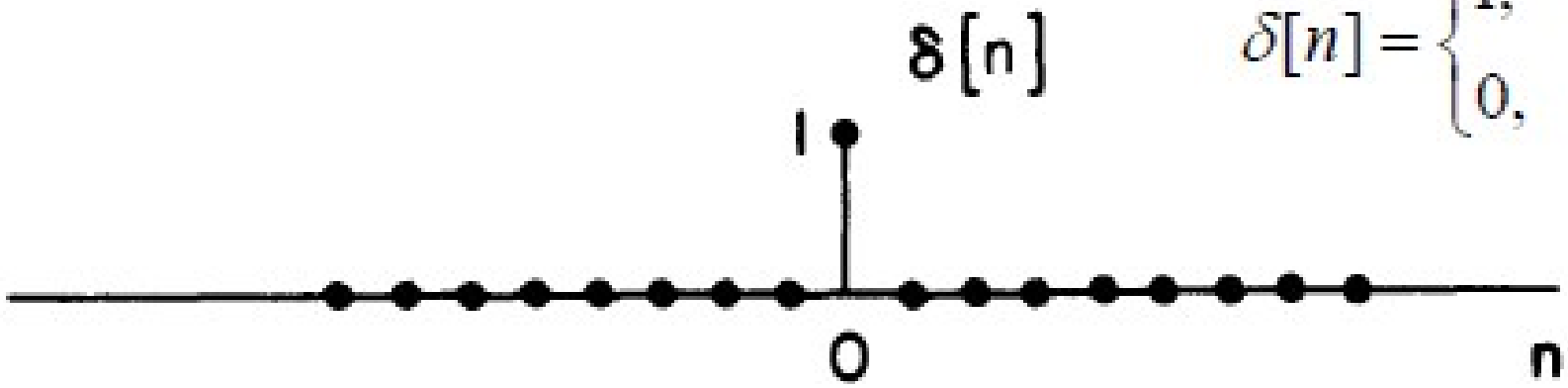


Unit-Ramp Function *r[n]:*



$$r[n] = \begin{cases} n, & n = 0, 1, \ldots \\ 0, & n = -1, -2, \ldots \end{cases}$$

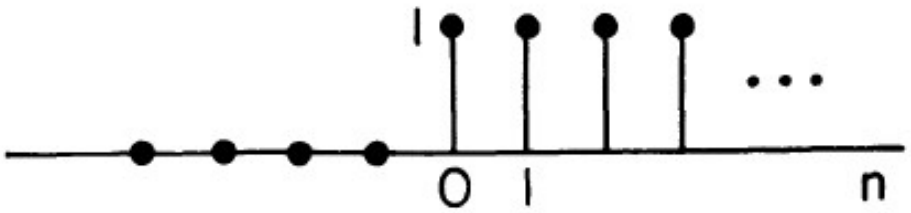
Unit Impulse Function δ[n]:

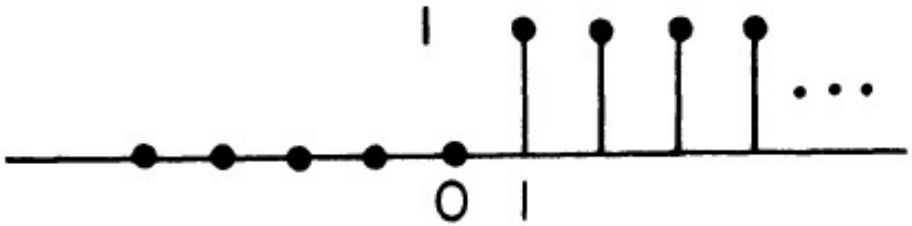


$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

What is the relationship between the discrete-time unit-step function *u[n] and the discrete-time unit-ramp function r[n]?*

What is the relationship between the discrete-time unit-step function *u[n] and the discrete-time unit-impulse function δ[n]?*

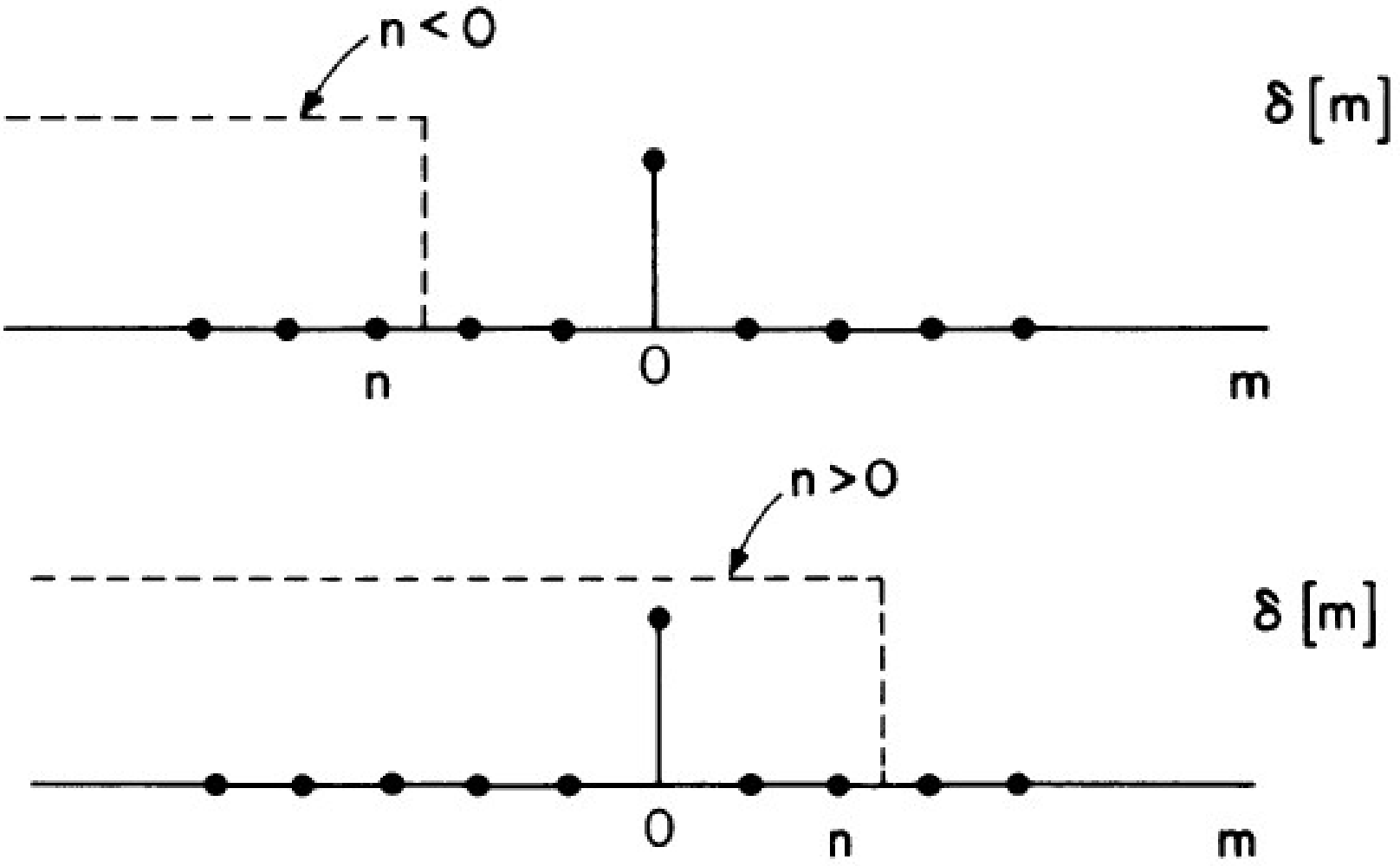






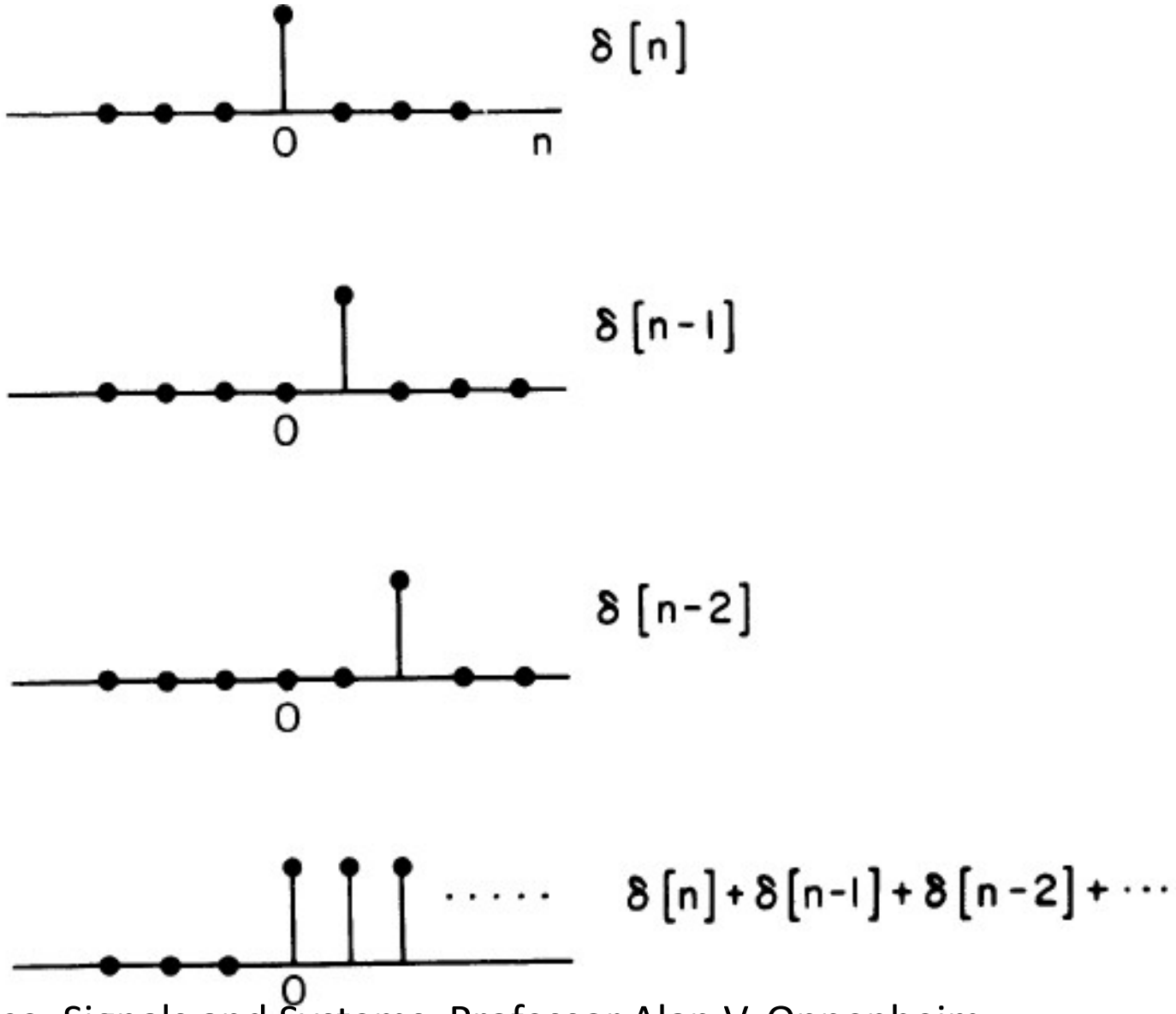
δ[n]=?

u[n]=?



Resource: Signals and Systems, Professor Alan V. Oppenheim

u[n]=?



Resource: Signals and Systems, Professor Alan V. Oppenheim

Discrete-time rectangular pulse:

$$p_L[n] = \begin{cases} 1, & n = -(L-1)/2, \ldots, -1, 0, 1, \ldots, (L-1)/2 \\ 0, & \text{all other } n \end{cases}$$

