EE 362: *Signals and Systems*

Exam 1
Spring 2019: February 21, 2019
Dr. Mohammed Ben-Idris

Print your name: Solutions

Student ID #: _____

Signature: _____
(By signing above, I am promising not to cheat on this exam.)
Exams with no signature will not be graded.

This exam is out of 400 points. Read through the exam before starting and feel free to do the easier parts first. Show your work. Careful of units.

Question	Score
1	/50
2	/145
3	/205
Total	/400

Problem #1: Matching & True/False 50 points

Match the **Property** in the left column to the **Phrase** in the right column by placing the appropriate letter from the Property column in the boxes provided. Each Property can be used any number of times, such that some letters might not be used, while others could be used more than once. (5 points each)

Property	**Phrase**
A) Linear	Derivative of a continuous-time unit-step function is [F].
B) Causal	[B] systems are non-anticipatory systems
C) Unit-step function	A system is said to be [E] if the input is shifted by time t1, the output is also shifted by the same time.
D) Unit-ramp function	Convolution of two continuous-time unit-step functions is [D].
E) Time-invariant	The output of [H] systems at any fixed time point depends only on the input $x(t)$.
F) Unit-impulse function	
G) Memoryless	
H) Non-recursive	

Answer true or false to the following statements by circling the correct answer (5 points per question).

True	(False)	The order in which parallel systems are connected is important.
(True)	False	Convolution of two unit-step discrete-time signals is n+1.
(True)	False	Linear systems must be additive and homogeneous.
True	(False)	Not all memoryless systems are causal systems.
True	(False)	In converting a complex number from rectangular form to polar form, we add/subtract 180^o if and only if the real part has a negative sign and the imaginary part has a positive sign.

Problem #2: 145 points

1. Consider the continuous-time signal shown in Figure 1: [55 points]

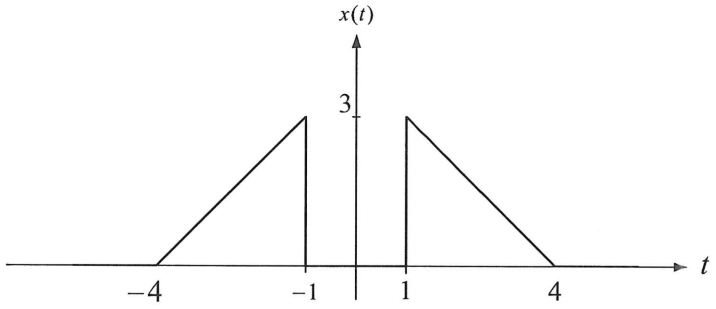


Figure 1. Problem 2 part 1

(a) express the signal in terms of rectangular pulses, $P_\tau(t)$, and/or triangular pulses, $\left(1-\frac{2|t|}{\tau}\right)P_\tau(t)$;

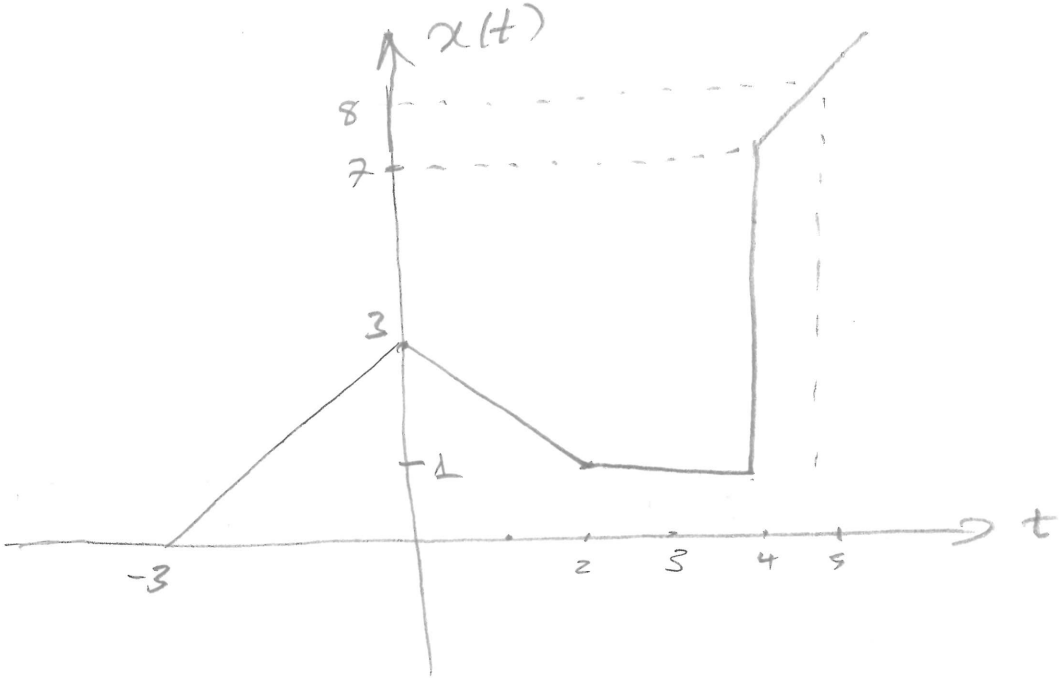
$$x(t) = 4\left(1-\frac{2|t|}{8}\right)P_8(t) - \left(1-\frac{2|t|}{2}\right)P_2(t) - 3P_2(t)$$

(b) express the signal in terms of unit-step and unit-ramp functions.

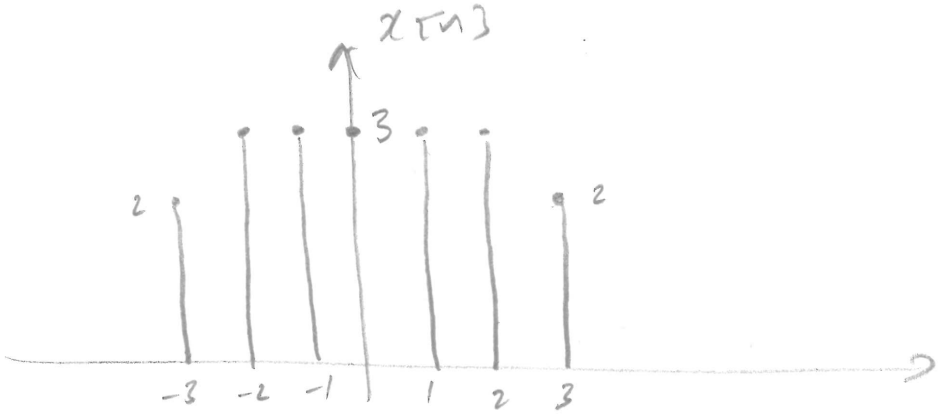
$$x(t) = r(t+4) - r(t+1) - 3u(t+1) + 3u(t-1) - r(t-1) + r(t-4)$$

2. Sketch the following functions:

(a) $x(t) = (t+3)u(t+3) - 2r(t) + r(t-2) + (t+2)u(t-4)$; [25 points]



(b) $x[n] = P_5[n] + 2P_7[n]$ [17 points]



3. For the following systems, determine whether they are linear, causal, time-invariant, and/or have memories. (answer YES or NO) [48 points]

System	Linear	Causal	Memoryless	Time-invariant
$y(t) = 2\cos(t)\,x(t)$	✓	✓	✓	X
$y(t) = 2x(t+4) + 3$	X	X	X	✓
$y[n] = 3n^2 x[n+2]$	✓	X	X	X
$y[n] = 4x[n-2]$	✓	✓	X	✓

Problem #3: 205 points

1) For the discrete-time signals $x[n]$ and $v[n]$ shown in Figure 2, compute the convolution $x[n] * v[n]$ and sketch the results. [50 points]

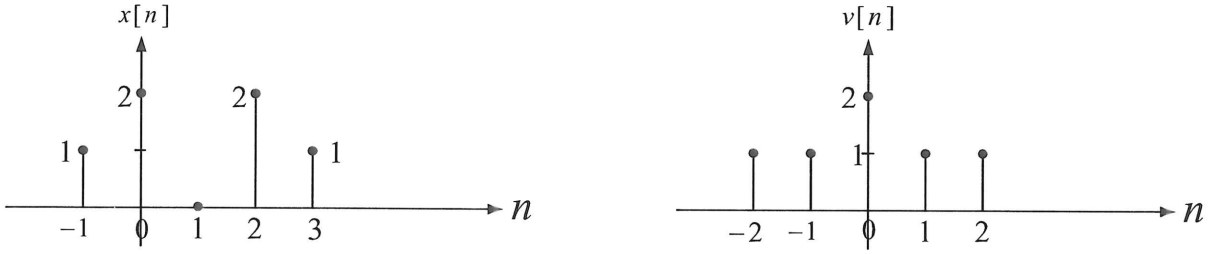
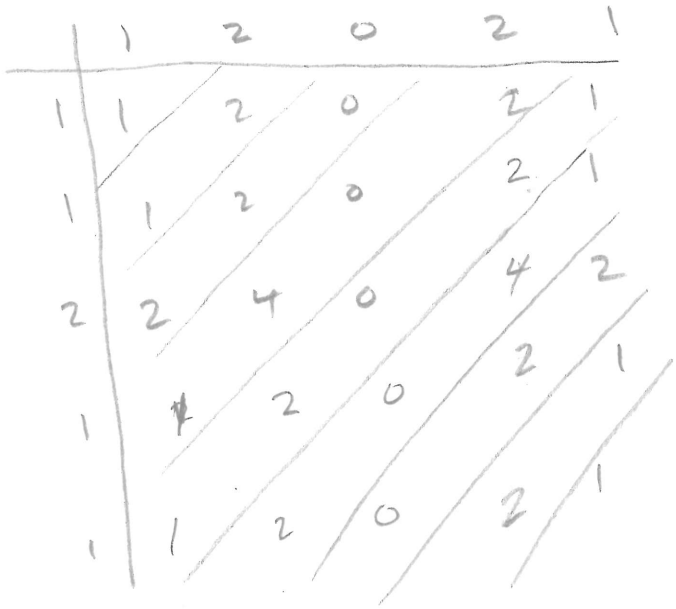
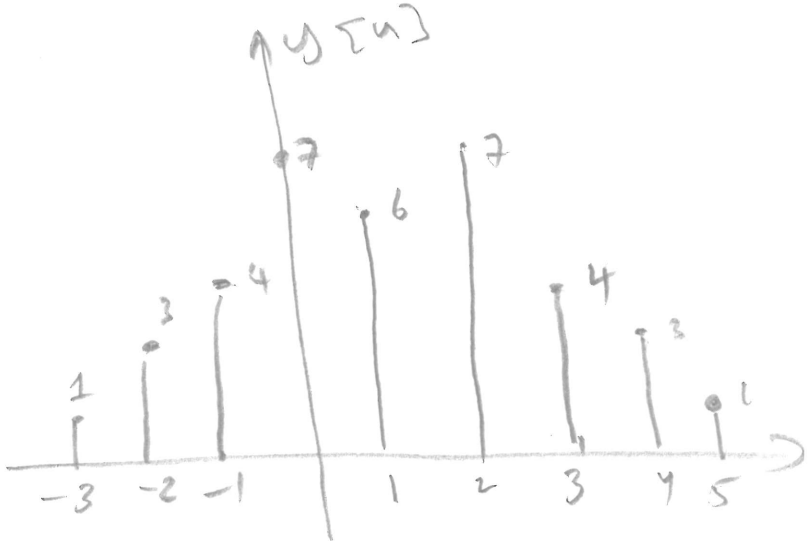


Figure 2. Problem 3 part 1





$y[-3] = 1$; $y[-2] = 3$; $y[-1] = 4$; $y[0] = 7$; $y[1] = 6$

$y[2] = 7$; $y[3] = 4$; $y[4] = 3$; $y[5] = 1$

$y[n] = 0$ for other values of n

2) For the following discrete-time signal, compute the unit-pulse response $h[n]$ for all integers $n \geq 0$. [25 points]

$$y[n+3] + 2y[n] = 2x[n+1]$$

$h[n+3] = -2h[n] + 2\delta[n+1]$

$h[0] = -2h[-3] + 2\delta[-2] = 0$

$h[1] = -2h[-2] + 2\delta[-1] = 0$

$h[2] = -2h[-1] + 2\delta[0] = 2$

$h[3] = -2h[0] + 2\delta[1] = 0$

$h[4] = -2h[1] + 2\delta[2] = 0$

$h[5] = -2h[2] + 2\delta[3] = -4$

$h[6] = h[7] = 0$

$h[8] = 8$

$h[9] = h[10] = 0$

$h[11] = -16$

$h[12] = h[13] = 0$

$h[14] = 32$; and so on

$\therefore h[n] = -(-2)^{\frac{n+1}{3}} = 2(-2)^{\frac{n-2}{3}}$

for $n = 2, 5, 8, 11, \cdots$

and $h[n] = 0$ O.W.

3) For the RLC circuit shown in Figure 3, find the input/output differential equation when the output is the voltage across the capacitor and the input is the voltage source. [50 points]

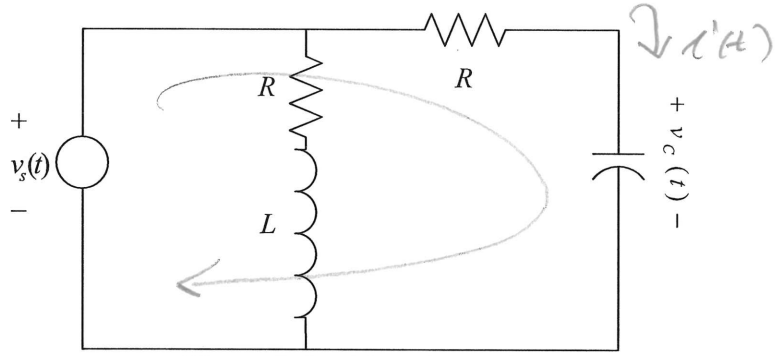


Figure 3. Problem 3 part 3

$$i(t) = i_c(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$$

$$-v_s(t) + R\,i(t) + v_c(t) = 0$$

OR

$$-x(t) + R\,i(t) + y(t) = 0$$

$$-x(t) + RC\frac{dy(t)}{dt} + y(t) = 0$$

OR

$$\boxed{\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)}$$

4) For the continuous-time signals $x(t)$ and $v(t)$ shown in Figure 4, compute the convolution $x(t) * v(t)$ for all $t \geq 0$ and sketch the results. [80 points]

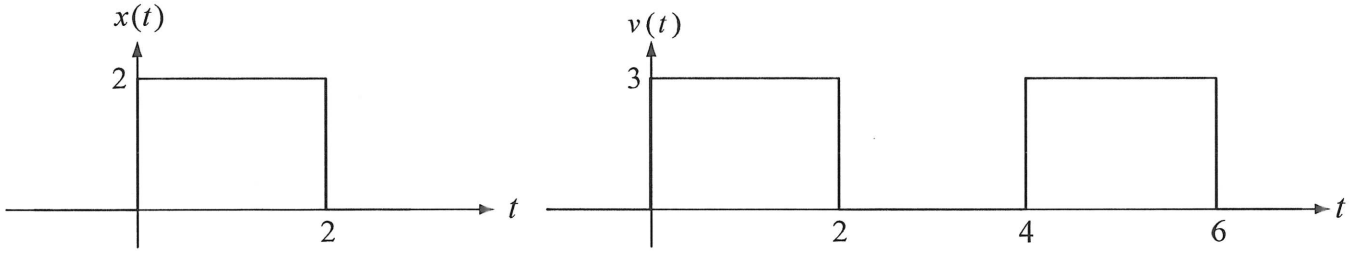
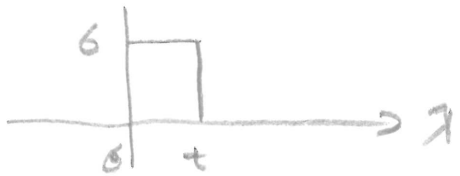


Figure 4. Problem 3 part 4

For $t \leq 0$ and $t \geq 8$, $x(t) * v(t) = 0$

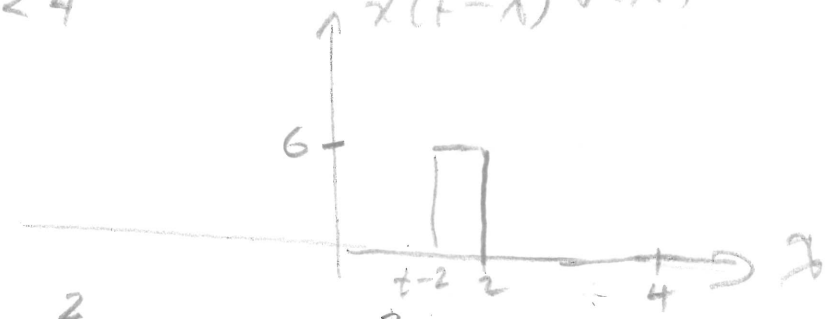
① For $0 \leq t < 2$

$x(t-\lambda)\,v(\lambda)$



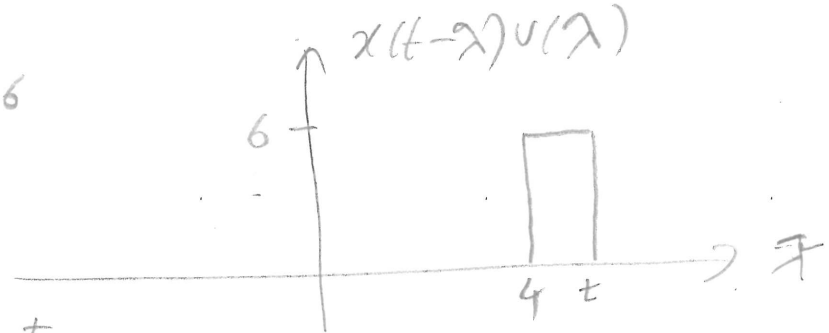
$$x(t) * v(t) = \int_0^t 6\,d\lambda = 6t$$

② For $2 \leq t < 4$



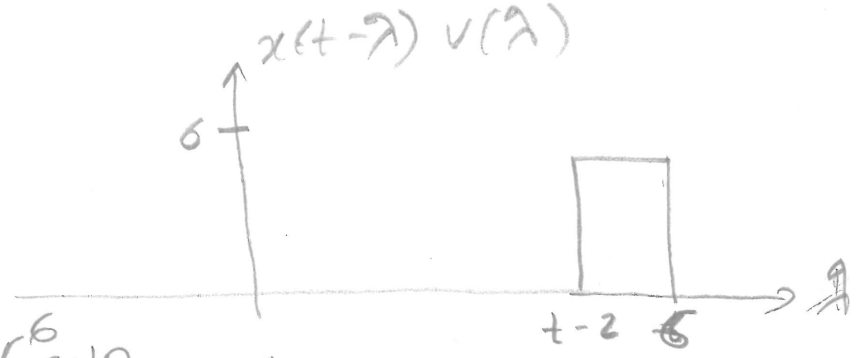
$$x(t) * v(t) = \int_{t-2}^{2} 6\,d\lambda = 6\lambda \Big|_{t-2}^{2} = 12 - 6t + 12 = -6t + 24$$

③ For $4 \leq t < 6$



$$x(t) * v(t) = \int_4^t 6\,d\lambda = 6t - 24$$

④ For $6 \le t < 8$



$$x(t) * v(t) = \int_{t-2}^{6} 6 \, d\lambda = 36 - 6t + 12 = -6t + 48$$

