EE 362: *Signals and Systems*

Exam 2
Spring 2019: April 4, 2019
Dr. Mohammed Ben-Idris

Print your name: solutions

Student ID #: _____

Signature: _____
(By signing above, I am promising not to cheat on this exam.)
Exams with no signature will not be graded.

This exam is out of 400 points. Read through the exam before starting and feel free to do the easier parts first. Show your work. Careful of units.

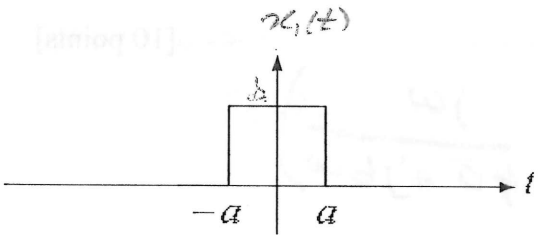
Question	Score
1	/35
2	/75
3	/290
Total	/400

Problem #1: Matching & True/False 35 points

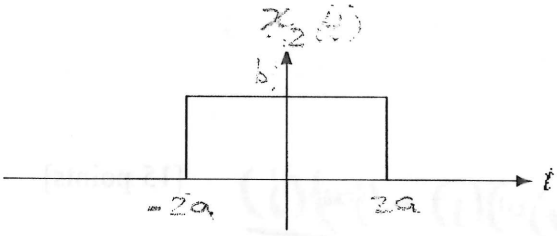
(a) Answer true or false to the following statements by circling the correct answer (4 points per question).

True	False	Time-limited signals cannot be bandlimited.
True	False	Trigonometric Fourier series can be applied only on periodic signals
True	False	Shorter-time-duration signals have more spectral content at higher frequencies than longer-time-duration signals.
True	False	Convolution in time domain is equivalent to multiplication in the frequency domain.
True	False	Fourier transform is always linear operation.

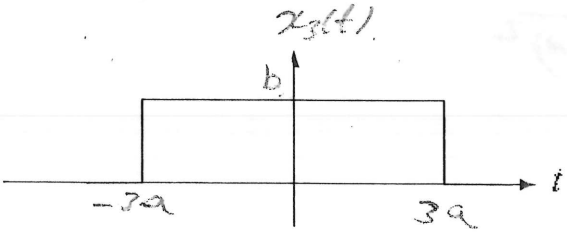
(a) Match the signals in the left column to their appropriate amplitude spectrums in the right column by placing the appropriate letter from the signals column in the boxes provided. [5 points per question]



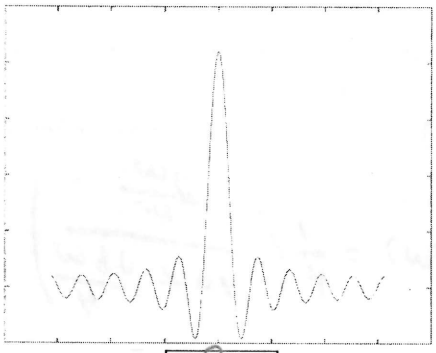
(A)



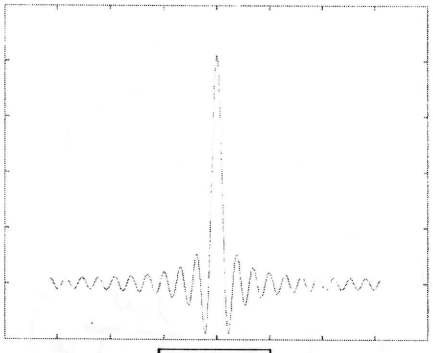
(B)



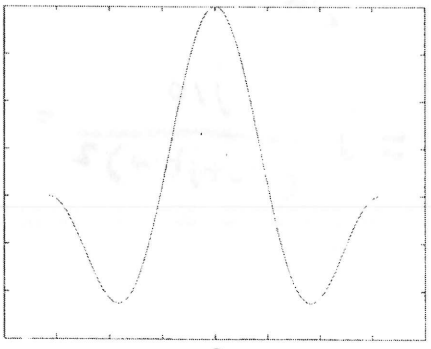
(C)



B



C



A

Problem #2: 75 points

A continuous-time signal $x(t)$ has the following Fourier transform where a and b are constants.

$$X(\omega) = \frac{j\omega}{10 + jb\omega}$$

1- Determine the Fourier transform $V(\omega)$ of the following signals:

1) $v(t) = x(t)\sin(10t)$ [14 points]

$$V(\omega) = \frac{j}{2}\left[\frac{j(\omega+10)}{10+j(\omega+10)} - \frac{j(\omega-10)}{10+j(\omega-10)}\right]$$

2) $v(t) = x(4t)$ [10 points]

$$V(\omega) = \frac{1}{4}\left(\frac{\frac{j\omega}{4}}{10+\frac{jb\omega}{4}}\right) = \frac{1}{4}\left(\frac{j\omega}{40+jb\omega}\right)$$

3) $v(t) = tx(t)$ [15 points]

$$V(\omega) = j\frac{d}{d\omega}\left(\frac{j\omega}{10+jb\omega}\right) = j\frac{(10+jb\omega)(j)-(j\omega)(jb)}{(10+jb\omega)^2}$$

$$= j\frac{j10}{(10+jb\omega)^2} = \frac{-10}{(10+jb\omega)^2}$$

B) A discrete-time signal $x[n]$ has DTFT as follows where a, b, and c are constants,

$$X(\Omega) = \frac{e^{-j\Omega}}{1 + be^{-j\Omega}}$$

Determine the DTFT $V(\Omega)$ of the following signals.

1) $v[n] = x[n + 4]$ [12 points]

$$V(\Omega) = \frac{e^{-j\Omega}}{1+be^{-j\Omega}} \cdot e^{j4\Omega} = \frac{e^{j3\Omega}}{1+be^{-j\Omega}}$$

2) $v[n] = x[-n]$ [12 points]

$$V(\Omega) = \frac{e^{j\Omega}}{1+be^{j\Omega}}$$

3) $v[n] = x[n]e^{j5n}$ [12 points]

$$V(\Omega) = \frac{e^{-j(\Omega-5)}}{1+be^{-j(\Omega-5)}}$$

Problem #3: 290 points

1) Compute the (sine/cosine) trigonometric Fourier series of the following periodic signal [70 points]

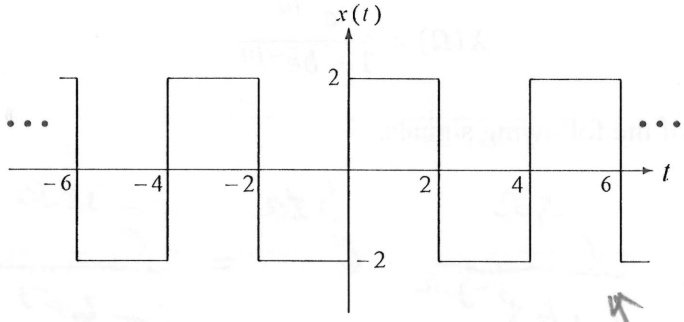


Figure 1. Problem 3 part 1

odd

$T = 4$; $\omega = \frac{\pi}{2}$; $a_0 = 0$; $a_k = 0$

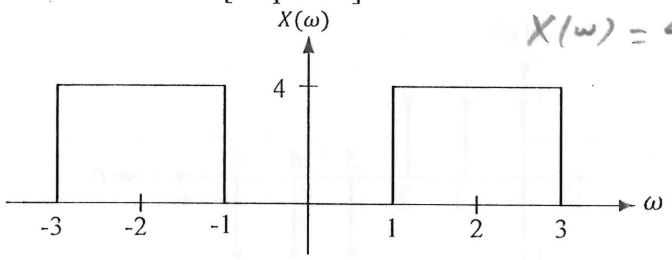
$$b_k = \frac{4}{4}\int_0^2 2\sin\left(\frac{k\pi}{2}t\right)dt = -\frac{2}{\frac{k\pi}{2}}\cos\left(\frac{k\pi}{2}t\right)\Big|_0^2$$

$$= -\frac{4}{k\pi}\left[\cos(k\pi) - 1\right] = \frac{4}{k\pi}\left[1 - \cos(k\pi)\right]$$

or $b_k = \begin{cases} \frac{8}{k\pi}, & k = 1, 3, 5, \ldots \\ 0, & k = 2, 4, 6, 8, \ldots \end{cases}$

$$x(t) = \sum_{k=1}^{\infty} \frac{4}{k\pi}\left[1 - \cos(k\pi)\right]\sin\left(\frac{k\pi}{2}t\right), \quad -\infty < t < \infty$$

2) Compute the inverse Fourier transform of the frequency function $X(\omega)$ shown below. You may use the duality concept to derive the transformation. [50 points]



$X(\omega) = 4P_2(t+2) + 4P_2(t-2)$

Figure 2. Problem 3 part 2

$4P_2(t) \leftrightarrow 8\,\text{sinc}\left(\frac{\omega}{\pi}\right)$

$8\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow 8\pi P_2(\omega)$

$\frac{4}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow 4P_2(\omega)$

$\frac{4}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) e^{-j2t} \leftrightarrow 4P_2(t+2)$

$\frac{4}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) e^{j2t} \leftrightarrow 4P_2(t-2)$

$\therefore x(t) = \frac{4}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\left[e^{j2t} + e^{-j2t}\right] = \frac{8}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\left[\frac{e^{j2t}+e^{-j2t}}{2}\right]$

$= \frac{8}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\cos(2t)$

3) Compute the Fourier transform of the following signal. [35 points]

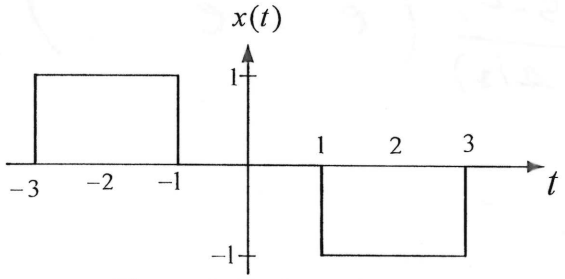


Figure 3. Problem 3 part 3

$x(t) = P_2(t+2) - P_2(t-2)$

$X(\omega) = 2\,\text{sinc}\left(\frac{\omega}{\pi}\right) e^{j2\omega} - 2\,\text{sinc}\left(\frac{\omega}{\pi}\right) e^{-j2\omega}$

$= 4j\,\text{sinc}\left(\frac{\omega}{\pi}\right)\left[\frac{e^{j2\omega} - e^{-j2\omega}}{2j}\right]$

$= 4j\,\text{sinc}\left(\frac{\omega}{\pi}\right)\sin(2\omega)$

4) Compute the Discrete-time Fourier Transform (DTFT) of the following continuous-time signal. [35 points]

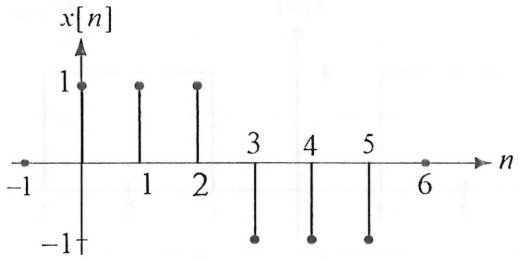


Figure 4. Problem 3 part 4

$$x[n] = P_2[n-1] - P_2[n-4]$$

$$X(\Omega) = \frac{\sin(1.5\Omega)}{\sin(\Omega/2)} e^{-j\Omega} - \frac{\sin(1.5\Omega)}{\sin(\Omega/2)} e^{-j4\Omega}$$

$$= \frac{\sin(1.5\Omega)}{\sin(\Omega/2)} \left(e^{-j\Omega} - e^{-j4\Omega} \right)$$

5) Compute the rectangular form of the four point DFT of the following signal: [65 points]
$x[0] = -2$, $x[1] = -2$, $x[2] = 2$, $x[3] = 2$, and $x[n] = 0$ for all other n.

$N = 4$

$$X_k = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi k}{4}n}$$

$$= -2e^{0} - 2e^{-j\frac{k\pi}{2}} + 2e^{-jk\pi} + 2e^{-j\frac{3k\pi}{2}}$$

$X_0 = -2 - 2 + 2 + 2 = 0$

$X_1 = -2 - 2e^{-j\frac{\pi}{2}} + 2e^{-j\pi} + 2e^{-j\frac{3\pi}{2}}$

$= -2 + 2j - 2 + 2j = -4 + j4$

$X_2 = -2 - 2e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi}$

$= -2 + 2 + 2 - 2 = 0$

$X_3 = -2 - 2e^{-j\frac{3\pi}{2}} + 2e^{-j3\pi} + 2e^{-j\frac{9\pi}{2}}$

$= -2 - j2 - 2 - j2 = -4 - j4$

$X_0 = 0$

$X_1 = -4 + j4$

$X_2 = 0$

$X_3 = -4 - j4$

6) If the output of the following circuit is the voltage across the inductor. Compute the output $y(t)$ for $-\infty < t < \infty$ when the input $x(t)$ is: $x(t) = 5\cos(200t + 3\pi/2)$ [35 points]
$R = 20\ (\Omega)$ and $L = 0.1$ (H)

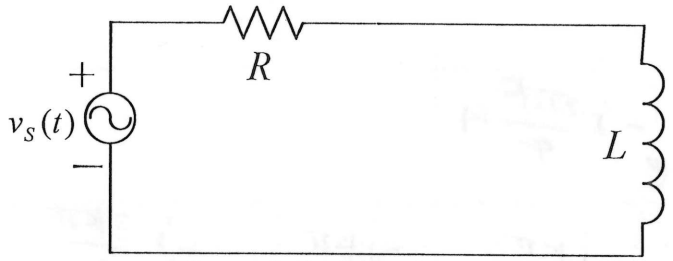
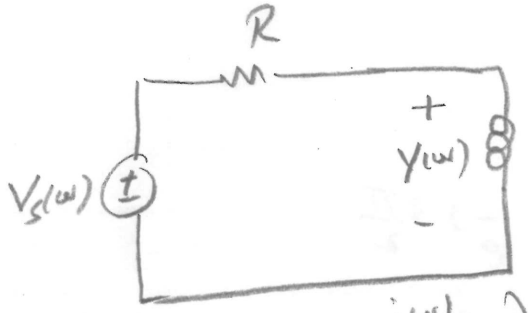


Figure 5. Problem 3 part 6



$$Y(\omega) = V_S(\omega)\left(\frac{j\omega L}{R+j\omega L}\right) \Rightarrow H(\omega) = \frac{j\omega L}{R+j\omega L}$$

$$= \frac{j\omega}{\frac{R}{L}+j\omega}$$

$$= \frac{j\omega}{200+j\omega}$$

$$H(200) = \frac{j200}{200+j200}$$

$$|H(200)| = \frac{1}{\sqrt{2}} \quad ; \quad \angle H(200) = 90 - \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$y(t) = \frac{5}{\sqrt{2}}\cos\left(200t + \frac{3\pi}{2} + \frac{\pi}{4}\right)$$

$$= \frac{5}{\sqrt{2}}\cos\left(200t + \frac{7\pi}{4}\right)$$

$$y(t) = 3.5355\cos(200t + 315^\circ)$$