Continuous-Time Fourier Series representation:

$$x(t) = a_0 + \sum_{k=1}^{N} [a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)] \qquad -\infty < t < \infty$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(k\omega_0 t)dt \qquad k = 1,2,...$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(k\omega_0 t)dt \qquad k = 1,2,...$$

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt = \begin{cases} \frac{2}{T}\int_0^{\frac{T}{2}} x(t)dt & \text{when } x(t) \text{ is } \underline{\text{even}} \\ 0 & \text{when } x(t) \text{ is } \underline{\text{odd}} \end{cases}$$

$$a_k = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(k\omega_0 t)dt = \begin{cases} \frac{4}{T}\int_0^{\frac{T}{2}} x(t)\cos(k\omega_0 t)dt & \text{when } x(t) \text{ is } \underline{\text{even}} \\ 0 & \text{when } x(t) \text{ is } \underline{\text{odd}} \end{cases}$$

$$b_k = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(k\omega_0 t)dt = \begin{cases} 0 & \text{when } x(t) \text{ is } \underline{\text{even}} \\ \frac{4}{T}\int_0^{\frac{T}{2}} x(t)\sin(k\omega_0 t)dt & \text{when } x(t) \text{ is } \underline{\text{odd}} \end{cases}$$

Continuous-Time Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad -\infty \le \omega \le \infty \quad , \quad X(\omega) = R(\omega) + jI(\omega)$$

$$R(\omega) = \begin{cases} 2\int_0^{\infty} x(t)\cos(\omega t)dt & \text{when } x(t) \text{ is } \underline{\text{even}} \\ 0 & \text{when x(t) is } \underline{\text{odd}} \end{cases}$$

$$I(\omega) = \begin{cases} 0 & \text{when } x(t) \text{ is } \underline{\text{even}} \\ 2\int_0^{\infty} x(t)\sin(\omega t)dt & \text{when } x(t) \text{ is } \underline{\text{odd}} \end{cases}$$

Continuous-Time inverse Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{-j\omega t}\,d\omega$$

Discrete-time Fourier Analysis

Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{jk\Omega_0 n}$ $a_k = \frac{1}{N} \sum_{n=<N>} x[n]\, e^{-jk\Omega_0 n}$
Discrete-Time Fourier Transform	$X(\Omega) = \sum_{n=-\infty}^{n=\infty} x[n] e^{-j\Omega n}$
Inverse DTFT	$x[n] = \frac{1}{2\pi} \int_0^{2\pi} X(\Omega) e^{j\Omega n} d\Omega$
Discrete Fourier Transform (DFT)	$X_k = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}} \qquad k = 0,1,2,\ldots,N-1$
Inverse DFT	$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X_k e^{j\frac{2\pi kn}{N}} \qquad n = 0,1,2,\ldots,N-1$

TABLE 3.1 Properties of the Fourier Transform

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Right or left shift in time	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right)\ a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by a power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n}X(\omega)\ n = 1, 2, \ldots$
Multiplication by a complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)\ \ \omega_0$ real
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin(\omega_0 t) \leftrightarrow \frac{j}{2}[X(\omega + \omega_0) - X(\omega - \omega_0)]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in the time domain	$\frac{d^n}{dt^n}x(t) \leftrightarrow (j\omega)^n X(\omega)\ n = 1, 2, \ldots$
Integration in the time domain	$\int_{-\infty}^{t} x(\lambda)\,d\lambda \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in the time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in the time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega)\,d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\,d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

TABLE 3.2 Common Fourier Transform Pairs

$1, -\infty < t < \infty \leftrightarrow 2\pi\delta(\omega)$

$-0.5 + u(t) \leftrightarrow \frac{1}{j\omega}$

$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

$\delta(t) \leftrightarrow 1$

$\delta(t - c) \leftrightarrow e^{-j\omega c}$, c any real number

$e^{-bt}u(t) \leftrightarrow \frac{1}{j\omega + b}$, $b > 0$

$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$, ω_0 any real number

$p_\tau(t) \leftrightarrow \tau\ \mathrm{sinc}\frac{\tau\omega}{2\pi}$

$\tau\ \mathrm{sinc}\frac{\tau t}{2\pi} \leftrightarrow 2\pi p_\tau(\omega)$

$\left(1 - \frac{2|t|}{\tau}\right)p_\tau(t) \leftrightarrow \frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$

$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau t}{4\pi}\right) \leftrightarrow 2\pi\left(1 - \frac{2|\omega|}{\tau}\right)p_\tau(\omega)$

$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$

$\cos(\omega_0 t + \theta) \leftrightarrow \pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$

$\sin(\omega_0 t) \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$

$\sin(\omega_0 t + \theta) \leftrightarrow j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$

TABLE 4.1 Common DTFT Pairs

$1, \text{all } n \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - 2\pi k)$

$\text{sgn}[n] \leftrightarrow \frac{2}{1 - e^{-j\Omega}}$, where $\text{sgn}[n] = \begin{cases} 1, & n = 0, 1, 2, \ldots \\ -1, & n = -1, -2, \ldots \end{cases}$

$u[n] \leftrightarrow \frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$

$\delta[n] \leftrightarrow 1$

$\delta[n - q] \leftrightarrow e^{-jq\Omega}, q = \pm 1, \pm 2, \ldots$

$a^n u[n] \leftrightarrow \frac{1}{1 - ae^{-j\Omega}}, \quad |a| < 1$

$e^{j\Omega_0 n} \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - \Omega_0 - 2\pi k)$

$p[n] \leftrightarrow \frac{\sin\left[\left(q + \frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$

$\frac{B}{\pi}\text{sinc}\left(\frac{B}{\pi}n\right) \leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$

$\cos \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[\delta(\Omega + \Omega_0 - 2\pi k) + \delta(\Omega - \Omega_0 - 2\pi k)]$

$\sin \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega + \Omega_0 - 2\pi k) - \delta(\Omega - \Omega_0 - 2\pi k)]$

$\cos(\Omega_0 n + \theta) \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[e^{-j\theta}\delta(\Omega + \Omega_0 - 2\pi k) + e^{j\theta}\delta(\Omega - \Omega_0 - 2\pi k)]$

TABLE 4.2 Properties of the DTFT

Property	Transform Pair/Property
Linearity	$ax[n] + bv[n] \leftrightarrow aX(\Omega) + bV(\Omega)$
Right or left shift in time	$x[n - q] \leftrightarrow X(\Omega)e^{-jq\Omega}$, q any integer
Time reversal	$x[-n] \leftrightarrow X(-\Omega) = \overline{X(\Omega)}$
Multiplication by n	$nx[n] \leftrightarrow j\frac{d}{d\Omega}X(\Omega)$
Multiplication by a complex exponential	$x[n]e^{jn\Omega_0} \leftrightarrow X(\Omega - \Omega_0)$, Ω_0 real
Multiplication by $\sin \Omega_0 n$	$x[n] \sin \Omega_0 n \leftrightarrow \frac{j}{2}[X(\Omega + \Omega_0) - X(\Omega - \Omega_0)]$
Multiplication by $\cos \Omega_0 n$	$x[n] \cos \Omega_0 n \leftrightarrow \frac{1}{2}[X(\Omega + \Omega_0) + X(\Omega - \Omega_0)]$
Convolution in the time domain	$x[n] * v[n] \leftrightarrow X(\Omega)V(\Omega)$
Summation	$\sum_{i=0}^{n} x[i] \leftrightarrow \frac{1}{1 - e^{-j\Omega}}X(\Omega) + \sum_{n=-\infty}^{\infty} \pi X(2\pi n)\delta(\Omega - 2\pi n)$
Multiplication in the time domain	$x[n]v[n] \leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega - \lambda)V(\lambda)\, d\lambda$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x[n]v[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \overline{X(\Omega)}V(\Omega)\, d\Omega$
Special case of Parseval's theorem	$\sum_{n=-\infty}^{\infty} x^2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(\Omega)\rvert^2\, d\Omega$
Relationship to inverse CTFT	If $x[n] \leftrightarrow X(\Omega)$ and $\gamma(t) \leftrightarrow X(\omega)p_{2\pi}(\omega)$, then $x[n] = \gamma(t)\lvert_{t=n} = \gamma(n)$