EE 362: Signals and Systems

Spring 2019

Exam 1: Sample Exam

Problem #1: Matching & True/False 15 points

Match the **Property** in the left column to the **Phrase** in the right column by placing the appropriate letter from the Property column in the boxes provided. Each Property can be used any number of times, such that some letters might not be used, while others could be used more than once. (1.5 points each)

Property	**Phrase**
A) Linear	[F] is zero for all nonzero values of t.
B) Causal	The output of [D] systems at any fixed time point depends only on the input $x(t)$.
C) Recursive	[G] Ideal time delay.
D) Non-recursive	[E] Ideal predictor.
E) y(t)=x(t+1)	A system is [A] if it is both additive and homogeneous.
F) Impulse function	
G) y(t)=x(t-1)	
H) Unit-step function	
I) Unit-ramp function	
J) Time-invariant	

Answer true or false to the following statements by circling the correct answer (1.5 points per question).

True	False	The slope of unit-ramp is 1 for t < 0.
True	False	Discrete-time systems can be represented by difference equations and continuous-time systems can be represented by differential equations.
True	False	The convolution technique can be used only for discrete-time signals.
True	False	Convolution of two unit-step continuous-time signals is a unit-ramp.
True	False	All memoryless systems are causal systems.

Problem #2: 35 points

1. Consider the continuous-time signal shown in Figure 1: [11 points]

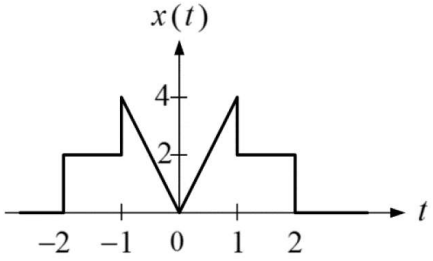


Figure 1. Problem 2 part 1

(a) express the signal in terms of rectangular pulses, $P_\tau(t)$, and/or triangular pulses, $\left(1-\frac{2|t|}{\tau}\right)P_\tau(t)$;

$$x(t) = 2P_4(t) + 2P_2(t) - 4\left(1-\frac{2|t|}{2}\right)P_4(t)$$

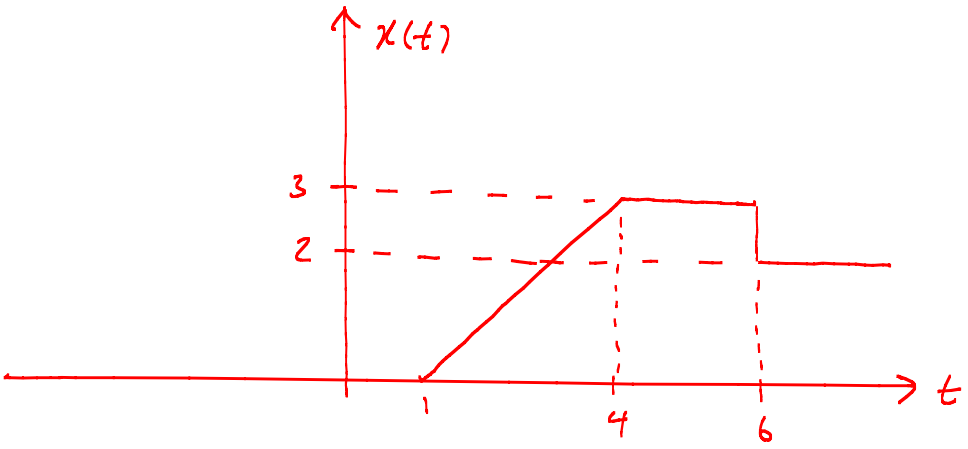
(b) express the signal in terms of unit-step and unit-ramp functions.

$$x(t) = 2u(t+2) + 2u(t+1) - 4r(t+1) + 8r(t) - 4r(t-1) - 2u(t-1) - 2u(t-2)$$

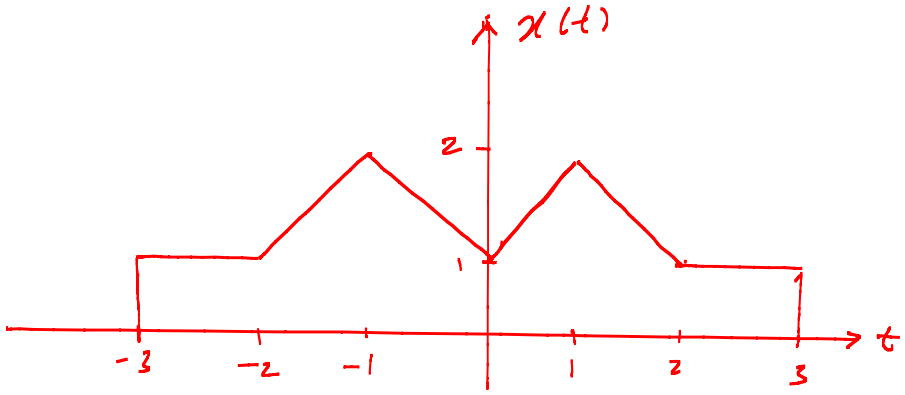
2. Sketch the following functions: [12 points]

(a) $x(t) = (t-1)u(t-1) - (t-4)u(t-4) - u(t-6)$;

$(t-1)u(t-1) = r(t-1)$; $(t-4)u(t-4) = r(t-4)$



(b) $x(t) = u(t+3) + r(t+2) - 2r(t+1) + 2r(t) - 2r(t-1) + r(t-2) - u(t-3)$.



3. For the following systems, determine whether they are linear, causal, time-invariant, and/or have memories. (answer YES or NO) [12 points]

System	Linear	Causal	Memoryless	Time-invariant
$y(t) = e^{-t}x(t)$	Yes	Yes	Yes	No
$y(t) = 2x^2(t+4)$	No	No	No	yes
$y[n] = 2x[n-1] + 2$	NO	Yes	NO	Yes
$y[n] = n\,x[n]$	Yes	Yes	Yes	NO

Problem #3: 50 points

1) For the discrete-time signals $x[n]$ and $v[n]$ shown in Figure 2, compute the convolution $x[n] * v[n]$ and sketch the results. [12 points]

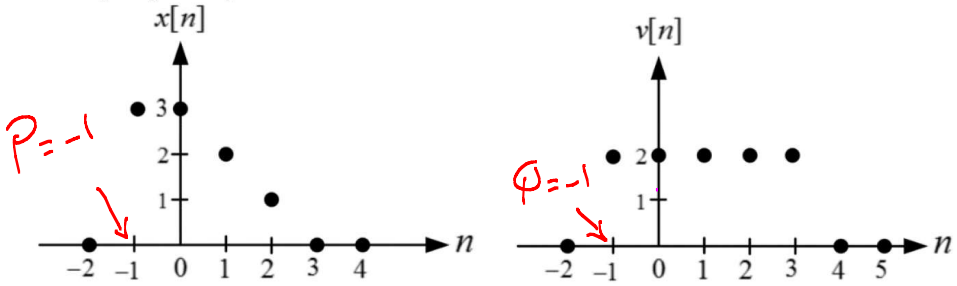
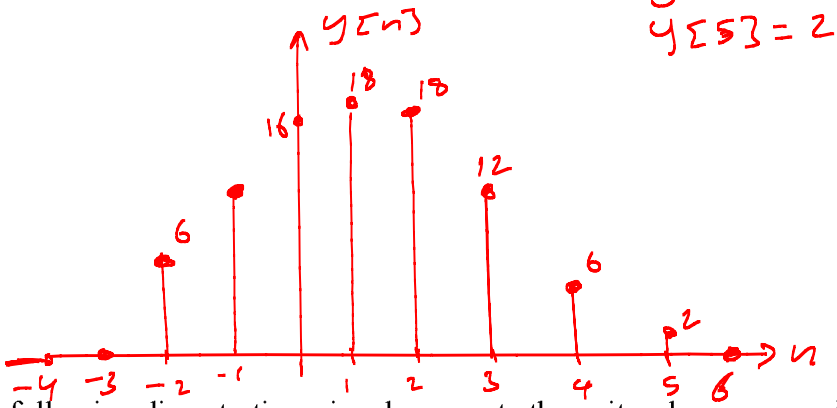


Figure 2. Problem 3 part 1

$y[P+Q+i] = ?$; $i = 0, 1, 2, \ldots$

	2	2	2	2	2
3	6	6	6	6	6
3	6	6	6	6	6
2	4	4	4	4	4
1	2	2	2	2	2

$y[-2] = 6$
$y[-1] = 12$
$y[0] = 16$
$y[1] = 18$
$y[2] = 18$
$y[3] = 12$
$y[4] = 6$
$y[5] = 2$



2) For the following discrete-time signal, compute the unit-pulse response $h[n]$ for all integers $n \geq 0$. [8 points]

$$y[n+2] + ay[n] = x[n]$$

$y[n+2] = -ay[n] + x[n]$
$h[n+2] = -ah[n] + \delta[n]$
$h[0] = -ah[-2] + \delta[-2] = 0$
$h[1] = -ah[-1] + \delta[-1] = 0$
$h[2] = -ah[0] + \delta[0] = 1$
$h[3] = -ah[1] + \delta[1] = 0$
$h[4] = -ah[2] + \delta[2] = -a$

$h[5] = -ah[3] + \delta[3] = 0$
$h[6] = -ah[4] + \delta[4] = a^2$
$h[7] = 0$; $h[8] = -a^3$; and so on

$$\therefore h[n] = \begin{cases} (-a)^{\frac{n}{2}-1} & ; \text{for } n = 2, 4, \ldots \\ 0 & ; \text{otherwise} \end{cases}$$

3) For the RLC circuit shown in Figure 3, find the input/output differential equation when the output is the current through the circuit and the input is the voltage source. [12 points]

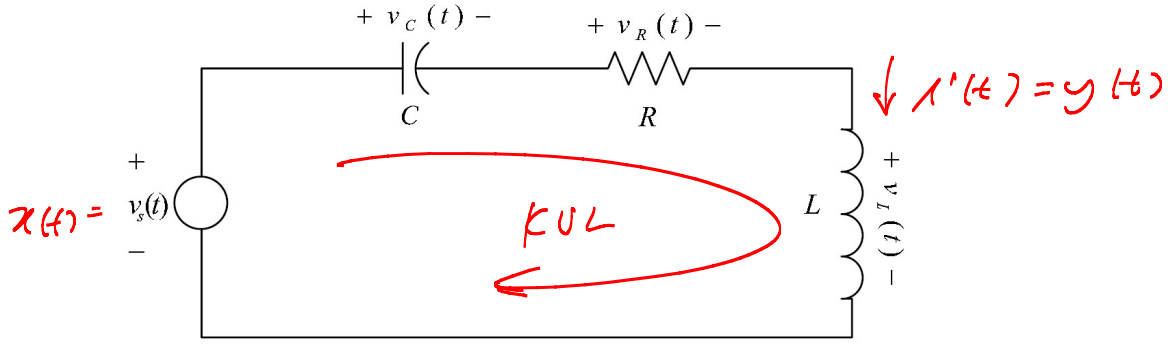


Figure 3. Problem 3 part 3

$-V_s(t) + V_C(t) + V_R(t) + V_L(t) = 0 \Rightarrow -x(t) + V_C(t) + V_R(t) + V_L(t) = 0$

$V_C(t) = \frac{1}{C}\int i'(t)\,dt = \frac{1}{C}\int y(t)\,dt$

$V_R(t) = R\,i'(t) = R\,y(t)$

$V_L(t) = L\,\frac{di'(t)}{dt} = L\,\frac{dy(t)}{dt}$

$\therefore -x(t) + \frac{1}{C}\int y(t)\,dt + R\,y(t) + L\,\frac{dy(t)}{dt} = 0$

$-\frac{dx(t)}{dt} + \frac{1}{C}y(t) + R\,\frac{dy(t)}{dt} + L\,\frac{d^2y(t)}{dt} = 0$

$\therefore \quad \frac{d^2y(t)}{dt^2} + \frac{R}{L}\,\frac{dy(t)}{dt} + \frac{1}{LC}\,y(t) = \frac{1}{L}\,\frac{dx(t)}{dt}$

4) For the continuous-time signals $x(t)$ and $v(t)$ shown in Figure 4, compute the convolution $x(t) * v(t)$ for all $t \geq 0$ and sketch the results. [18 points]

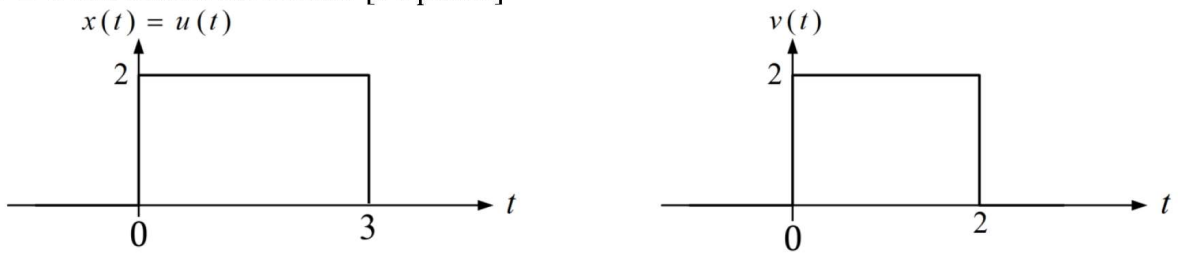
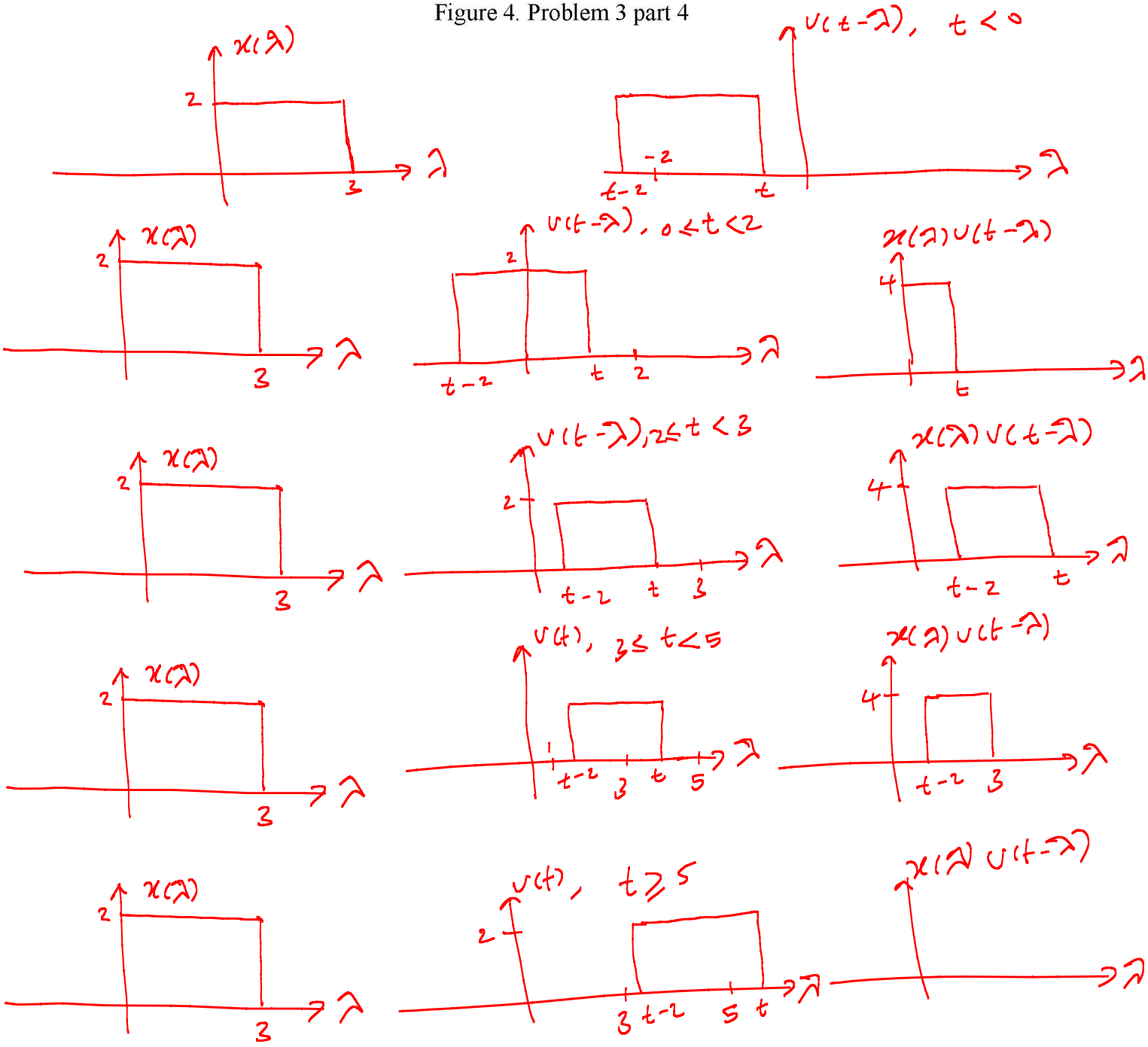


Figure 4. Problem 3 part 4



$x(t) * v(t) = 0$ for $t<0$ 或 $t \geq 5$

for $0 \leq t < 2$

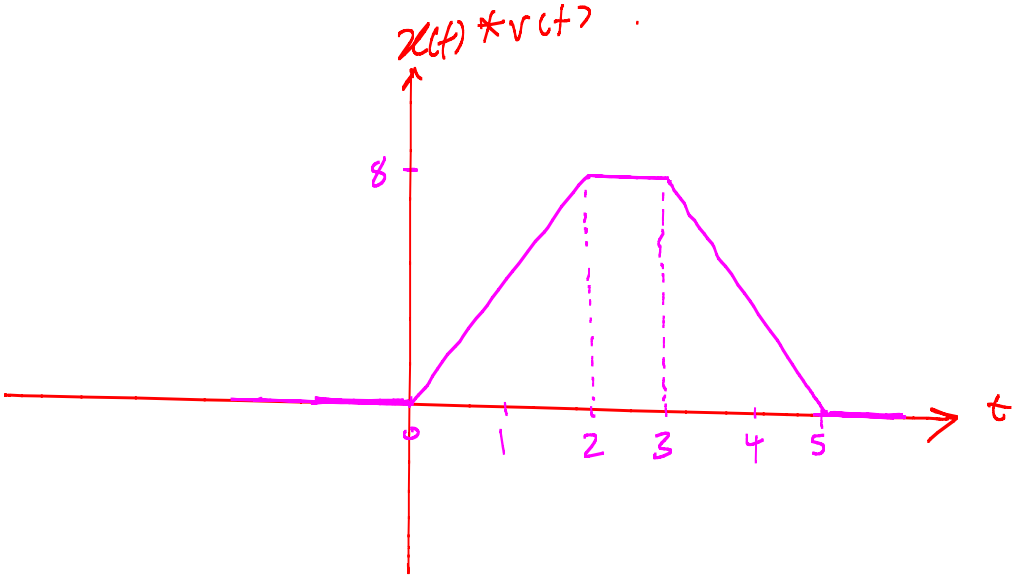
$$x(t) * v(t) = \int_0^t 4\lambda \, d\lambda = 4t$$

for $2 \leq t < 3$

$$x(t) * v(t) = \int_{t-2}^t 4\lambda \, d\lambda = 8$$

for $3 \leq t < 5$

$$x(t) * v(t) = \int_{t-2}^3 4\lambda \, d\lambda = -4t + 20$$



Convolution of two discrete-time signals $x[n]$ and $v[n]$ is defined as

$$x[n] * v[n] = \sum_{i=-\infty}^{\infty} x[i]v[n-i]$$

Convolution of two continuous-time signals $x(t)$ and $v(t)$ is defined as:

$$x(t) * v(t) = \int_{-\infty}^{\infty} x(t)v(t-\lambda)d\lambda$$

N-point moving average (MA) filter

$$y[n] = \sum_{i=0}^{N-1} \frac{1}{N} x[n-i]$$

N-Point Exponentially Weighted Moving Average (EWMA) Filter

$$y[n] = \sum_{i=0}^{N-1} a\left(b^i x[n-i]\right)$$

Current through a capacitor:

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

Voltage across an inductor:

$$v_L(t) = L\frac{di_L(t)}{dt}$$