Note Title 5/3/2018

$u[n] \leftrightarrow \frac{z}{z-1}$

$\cos(\Omega n)u[n] \leftrightarrow \frac{1}{2}\left[\frac{z e^{j\Omega}}{z e^{j\Omega}-1} + \frac{z e^{-j\Omega}}{z e^{-j\Omega}-1}\right]$

$\leftrightarrow \frac{1}{2}\left[\frac{z e^{j\Omega}(z e^{-j\Omega}-1) + z e^{-j\Omega}(z e^{j\Omega}-1)}{(z e^{j\Omega}-1)(z e^{-j\Omega}-1)}\right]$

$\leftrightarrow \frac{1}{2}\left[\frac{z^2 - z e^{j\Omega} + z^2 - z e^{-j\Omega}}{z^2 - z e^{j\Omega} - z e^{-j\Omega} + 1}\right]$

$\Leftrightarrow \left[\frac{z^2 - z\left(\frac{e^{j\Omega}+e^{-j\Omega}}{2}\right)}{z^2 - 2z\left(\frac{e^{j\Omega}+e^{-j\Omega}}{2}\right)+1}\right]$

$\leftrightarrow \frac{z^2 - z\cos(\Omega)}{z^2 - 2z\cos(\Omega)+1}$

$u[n] \leftrightarrow \frac{z}{z-1}$

$$
\begin{array}{r|l}
z-1 & z \\
 & z-1 \\
\hline
 & 1 \\
 & 1 - z^{-1} \\
\hline
 & z^{-1} \\
 & z^{-1} - z^{-2} \\
\hline
 & z^{-2} + \cdots
\end{array}
\qquad 1 + z^{-1} + z^{-2} + z^{-3} + \cdots
$$

$$z^{-2} - z^{-3}$$

$$z^{-3}$$

$$\vdots$$

$$\therefore X(z) = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= z^{0} + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= x[0]z^{0} + x[1]z^{-1} + x[2]z^{-2} + x[3]z^{-3} + \cdots$$

$$x[0] = 1,\ x[1] = 1,\ x[2] = 1,\ x[3] = 1,\ \cdots$$

$$\therefore \boxed{x[n] = u[n]}$$

Ex $X(z) = \dfrac{z^2+1}{z^2-1}$

$$z^2 - 1 \;\overline{)\; z^2 + 1}$$

$$1 + 2z^{-2} + 2z^{-4} + \cdots$$

$$z^2 - 1$$

$$2$$

$$2 - 2z^{-2}$$

$$2z^{-2}$$

$$\vdots$$

$$x[0] = 1,\ x[1] = 0,\ x[2] = 2,\ x[3] = 0$$

$$x[n] = 2,\ \cdots$$

$$\frac{X(z)}{z} = \frac{z^2+1}{z(z^2-1)}, \quad p_1 = 0,\ p_2 = 1,\ p_3 = -1$$

$$\frac{X(z)}{z} = \frac{c_0}{z} + \frac{c_1}{z-1} + \frac{c_2}{z+1} \qquad \left| \frac{X(z)}{z} = \frac{z^2+1}{z(z+1)(z-1)} \right.$$

$$c_0 = z\left(\frac{X(z)}{z}\right)\Big|_{z=0} = -1$$

$$c_1 = (z-1)\left(\frac{X(z)}{z}\right)\Big|_{z=1} = 1$$

$$c_2 = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = 1$$

$$a^n u[n] \leftrightarrow \frac{z}{z-a}$$

$$X(z) = \frac{c_0 z}{z} + \frac{c_1 z}{z-1} + \frac{c_2 z}{z+1} \qquad (-a)^n u[n] \leftrightarrow \frac{z}{z+a}$$

$$= 1 + \frac{z}{z-1} + \frac{z}{z+1}$$

$$\therefore\ x[n] = c_0 \delta[n] + c_1 (1)^n u[n] + c_2 (-1)^n u[n]$$

$$x[n] = -\delta[n] + u[n] + (-1)^n u[n]$$

$$x[0] = 1, \quad x[1] = 0, \quad x[2] = 2, \quad x[3] = 0$$

$$x[n] = 2, \quad x[5] = 0, \quad x[6] = 2, \ldots$$