Note Title 5/7/2018

$\delta[n] \leftrightarrow 1$

$u[n] \leftrightarrow \frac{z}{z-1}$

$a^n u[n] \leftrightarrow \frac{z}{z-a} \Rightarrow (5)^n u[n] \leftrightarrow \frac{z}{z-5}$ $\quad \not\Leftarrow (-5)^n u[n] \leftrightarrow \frac{z}{z+5}$

$u[n] - u[n-2] \leftrightarrow \frac{z}{z-1} - \frac{z}{z-1} z^{-2} = \frac{z - z^{-1}}{(z-1)}$

$n a^n u[n] \leftrightarrow -z \frac{d}{dz}\left(\frac{z}{z-a}\right) = -z\left(\frac{(z-a)(1) - (z)(1)}{(z-a)^2}\right)$

$\leftrightarrow -z\left(\frac{-a}{(z-a)^2}\right) = \frac{az}{(z-a)^2}$

$y[n] - 2y[n-1] = x[n]$, $y[-1] = 0$ Find $H(z)$

$Y(z) - 2z^{-1}Y(z) = X(z)$

$Y(z)\left(1 - 2z^{-1}\right) = X(z) \Rightarrow H(z) = \frac{1}{1 - 2z^{-1}} = \frac{z}{z-2}$

$X(z) = \frac{z}{1 - 2z^{-2}}$

$8z^2 X(z) - 2zX(z) - X(z) = z$

$8zX(z) - 2X(z) - z^{-1}X(z) = 1$

$$8x[n+1] - 2x[n] - x[n-1] = 1$$

$$8x[n] - 2x[n-1] - x[n-2] = 1$$

$$v[n] = x[n-4]u[n-4] \qquad X(z) = \frac{z}{8z^2 - 2z - 1}$$

$$V(z) = z^{-4}\left(\frac{z}{8z^2 - 2z - 1}\right) = \frac{z^{-3}}{8z^2 - 2z - 1}$$

$$v[n] = e^{3n}x[n]$$

$$a^n x[n] \longleftrightarrow X(z/a) \quad \Rightarrow \quad \boxed{a = e^3}$$

$$e^{3n}x[n] \longleftrightarrow \frac{z/e^3}{8(z/e^3)^2 - 2(z/e^3) - 1}$$

$$v[n] = x[n-3]u[n-3]$$

$$V(z) = z^{-3}\frac{z}{8z^2 - 2z - 1}$$

$$X(z) = \frac{1}{z - 1}$$

$$X(z) = \frac{z}{8z^2 - 2z - 1}$$

$v[n] = \cos(\Omega n)\, x[n] \Rightarrow \Omega = \frac{\pi}{3}$

$$V(z) = \frac{1}{2}\left[X(e^{j\Omega} z) + X(e^{-j\Omega} z)\right]$$

$$V(z) = \frac{1}{2}\left[\frac{z e^{j\frac{\pi}{3}}}{8z^2 e^{j\frac{\pi}{3}} - 2z e^{j\frac{\pi}{3}} - 1} + \frac{z e^{-j\frac{\pi}{3}}}{1 - 2z e^{-j\frac{\pi}{3}} - 8z^2 e^{-j\frac{\pi}{3}}}\right]$$

$$X(z) = \frac{z}{z+2}$$

$n \cos(\Omega n)\, x[n]$

$$n\, x[n] \longleftrightarrow -z\,\frac{(z+2)(1) - z(1)}{(z+2)^2} = \frac{-2z}{(z+2)^2}$$

$$j\, n \cos(\Omega n)\, x[n] \longleftrightarrow \frac{1}{2}\left[\frac{-2(z e^{j\frac{\pi}{3}})}{(z e^{j\frac{\pi}{3}} + 2)^2} + \frac{-2(z e^{-j\frac{\pi}{3}})}{(z e^{-j\frac{\pi}{3}} + 2)^2}\right]$$

$$X(z) = \frac{z}{8z^2 - 2z - 1}$$

$$x[0] = \lim_{z\to\infty} X(z) = \lim_{z\to\infty}\left(\frac{z}{1 - 2z - 8z^2}\right)$$

$$= \lim_{z\to\infty} \frac{1/z}{8 - 2/z - 1/z^2} = 0$$

$x[\infty] = ?$ Poles of $8z^2 - 2z - 1$ are: $\frac{1}{2}, -\frac{1}{4}$

$$\therefore x[\infty] = \lim_{z \to 1} (z-1)X(z) = \left. \frac{(z-1)(z)}{8z^2 - 2z - 1} \right|_{z=1} = 0$$

$$X(z) = \frac{z^2}{2z^3 + 3z^2 + 8z - 9}$$ you can choose to divide by z

$$X(z) = \frac{z}{2z^3 + 3z^2 + 8z - 9}$$ "

$$X(z) = \frac{1}{2z^3 + 3z^2 + 8z - 9}$$ " but it is better not to

$$\frac{X(z)}{z} = \frac{1}{2z^4 + 3z^3 + 8z^2 - 9z}$$ you can but complecates the problem

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Ex $$X(z) = \frac{2z}{z^2 + 5z + 6}$$

(1) $$\frac{X(z)}{z} = \frac{2}{(z+2)(z+3)} = \frac{A}{z+2} + \frac{B}{z+3}$$

$$A = (z+2)\left(\frac{z}{(z+2)(z+3)}\right)\Big|_{z=-2} = \frac{z}{-2+3} = 2$$

$$B = (z+3)\left(\frac{z}{(z+2)(z+3)}\right)\Big|_{z=-3} = \frac{z}{-3+2} = -2$$

$$\therefore \frac{X(z)}{z} = \frac{2}{z+2} - \frac{2}{z+3} \qquad a^n u[n] \leftrightarrow \frac{z}{z-a}$$

$$\therefore X(z) = 2\left(\frac{z}{z+2}\right) - 2\left(\frac{z}{z+3}\right)$$

$$\therefore x[n] = 2(-2)^n u[n] - 2(-3)^n u[n]$$

(2)

$$X(z) = \frac{A}{(z+2)} + \frac{B}{(z+3)}$$

$$A = (z+2)\left(\frac{2z}{(z+2)(z+3)}\right)\Big|_{z=-2} = \frac{2(-2)}{-2+3} = -4$$

$$B = (z+3)\left(\frac{2z}{(z+2)(z+3)}\right)\Big|_{z=-3} = \frac{2(-3)}{-3+2} = 6$$

$$\therefore X(z) = \frac{-4}{z+2} + \frac{6}{z+3}$$

$$X(z) = \left(\frac{-4z}{z+2}\right)z^{-1} + \left(\frac{6}{z+3}\right)z^{-1}$$

$$\therefore x[n] = -4(-2)^{n-1}u[n-1] + 6(-3)^{n-1}u[n-1] \quad \leftarrow \text{Solution (2)}$$

	Solution (1)	Solution (2)	
$n=0$	0	0	✓
$n=1$	2	2	✓
$n=2$	-10	-10	✓
⋮	⋮		

$$\therefore x[n] = 2(-2)^n u[n] - 2(-3)^n u[n] \quad \leftarrow \text{Solution (1)}$$

$$X(z) = \frac{2}{z^2+5z+6} = \frac{2}{(z+2)(z+3)}$$

$$\frac{X(z)}{z} = \frac{2}{z(z+2)(z+3)}$$