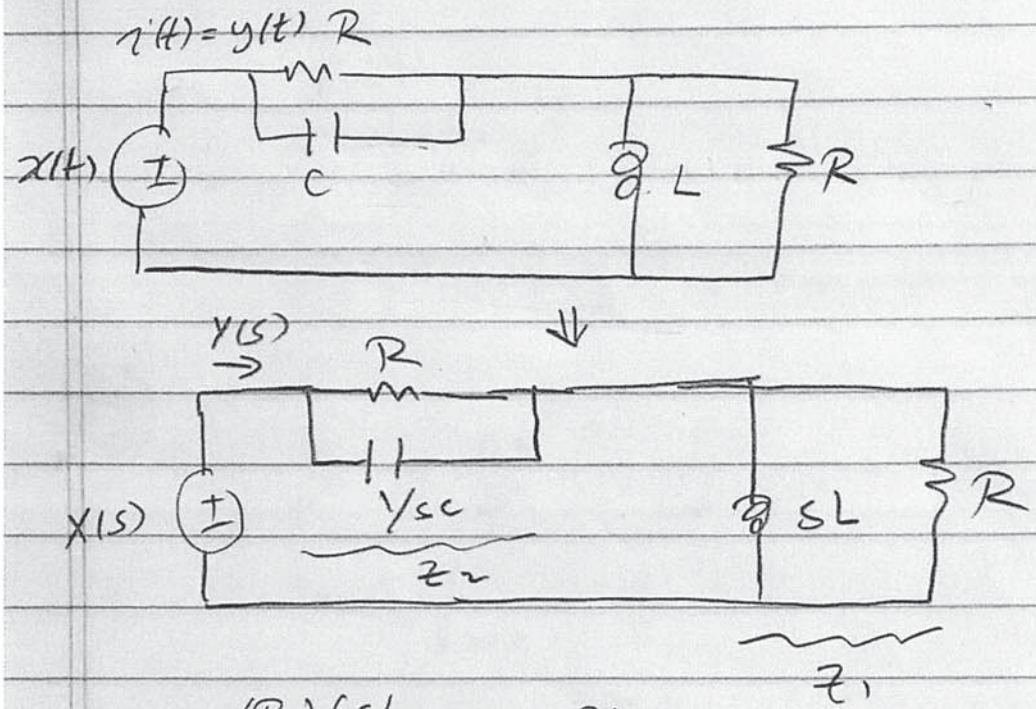
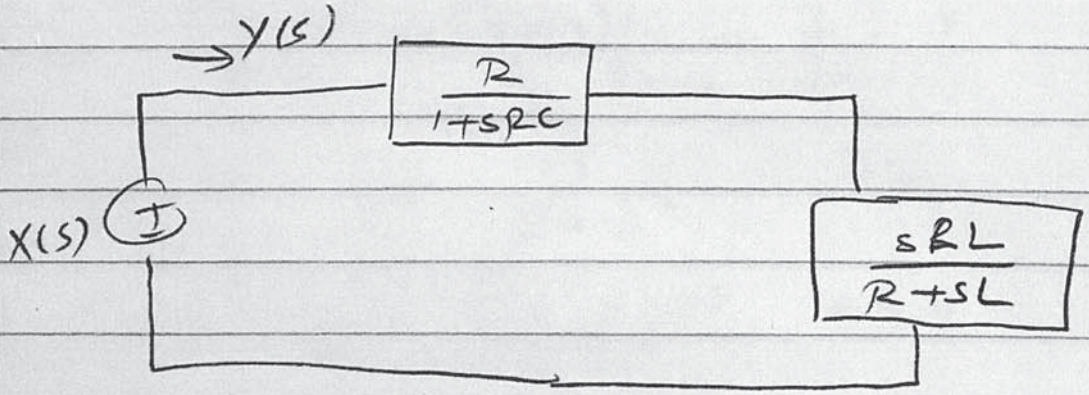
Laplace

① Find the transfer function of the following system



$$Z_1 = \frac{(R)(sL)}{R+sL} = \frac{sRL}{R+sL}$$

$$Z_2 = \frac{(R)(1/sC)}{R+1/sC} = \frac{R}{1+sRC}$$



$$Y(s) = \frac{X(s)}{\frac{R}{1+sRC} + \frac{sRL}{R+sL}} = X(s)\,\frac{(1+sRC)(R+sL)}{R(R+sL)+sRL(1+sRC)}$$

$$H(s) = \frac{R + s(R^2C+L) + s^2RLC}{R^2 + 2sRL + s^2R^2LC}$$

(3) Find the inverse Laplace transform [4]

$$X(s) = \frac{s+1}{s^4+4s^3+4s^2}$$

$$X(s) = \frac{s+1}{s^2(s^2+4s+4)} = \frac{s+1}{s^2(s+2)^2}$$

$$X(s) = \frac{c_1}{s^2} + \frac{c_2}{s} + \frac{c_3}{(s+2)^2} + \frac{c_4}{(s+2)}$$

$$c_1 = [s^2 X(s)]\Big|_{s=0} = \frac{1}{4}$$

$$c_2 = \left[\frac{d}{ds}(s^2 X(s))\right]\Big|_{s=0} = \frac{(s+2)^2 - (s+1)(2(s+2))}{(s+2)^4}\Big|_{s=0} = 0$$

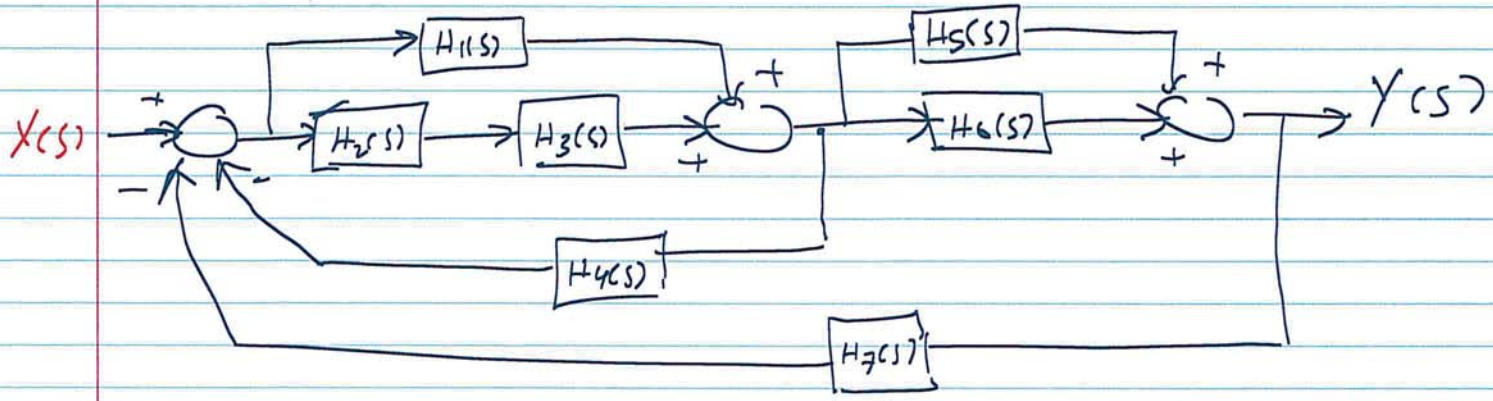
$$c_3 = [(s+2)^2 X(s)]\Big|_{s=-2} = \frac{s+1}{s^2}\Big|_{s=-2} = -\frac{1}{4}$$

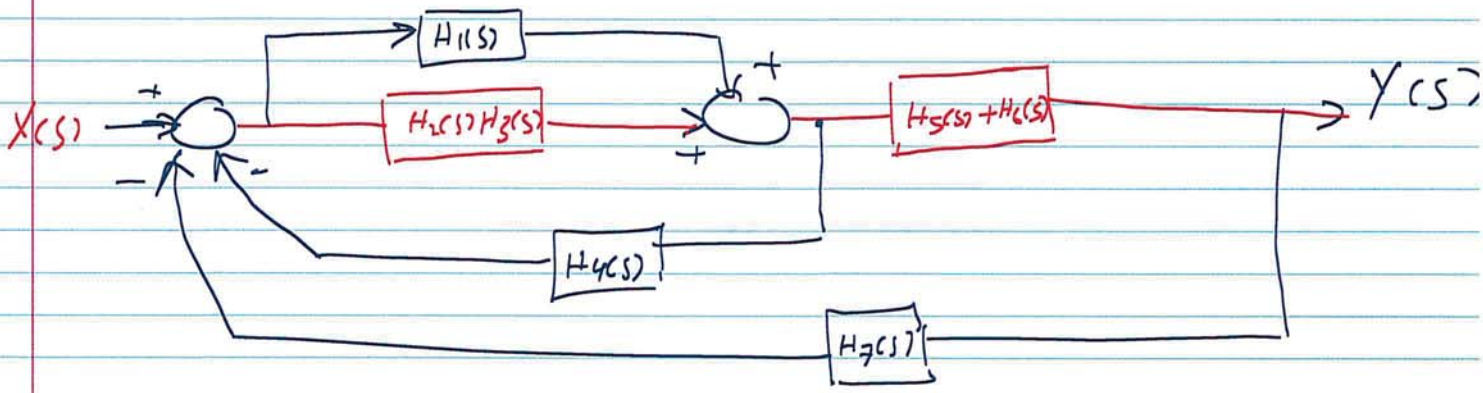
$$c_4 = \left[\frac{d}{ds}((s+2)^2 X(s))\right]_{s=-2} = \frac{d}{ds}\left(\frac{s+1}{s^2}\right)\Big|_{s=-2} = 0$$

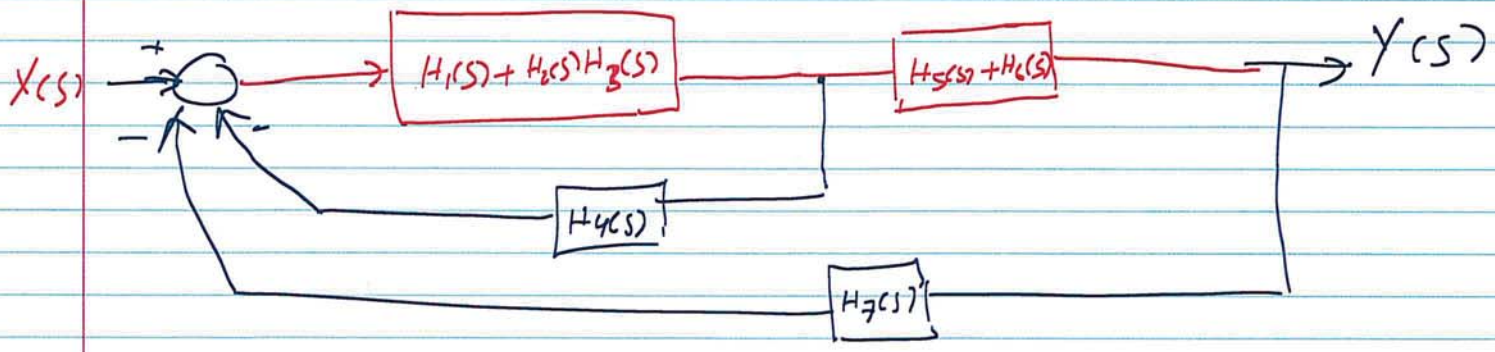
$$\Rightarrow X(s) = \frac{1/4}{s^2} - \frac{1/4}{(s+2)^2}$$

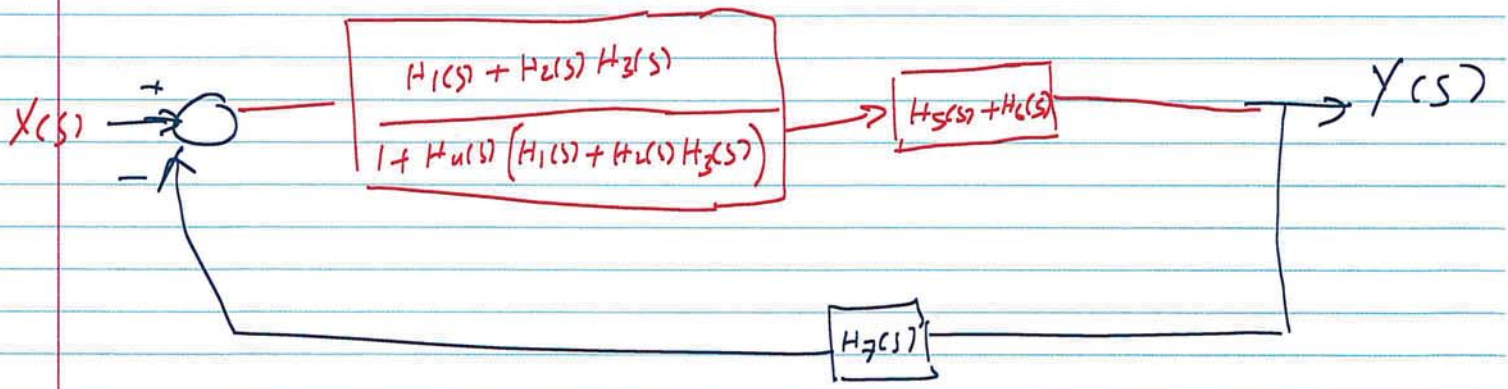
$$\therefore x(t) = \frac{1}{4}t - \frac{1}{4}te^{-2t}, \quad t \geq 0$$

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$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{(H_1+H_2H_3)(H_5+H_6)}{1+H_4(H_1+H_2H_3)}}{1+H_7\frac{(H_1+H_2H_3)(H_5+H_6)}{1+H_4(H_1+H_2H_3)}}$$

$$= \frac{(H_1+H_2H_3)(H_5+H_6)}{1+H_4(H_1+H_2H_3)+H_7(H_1+H_2H_3)(H_5+H_6)}$$

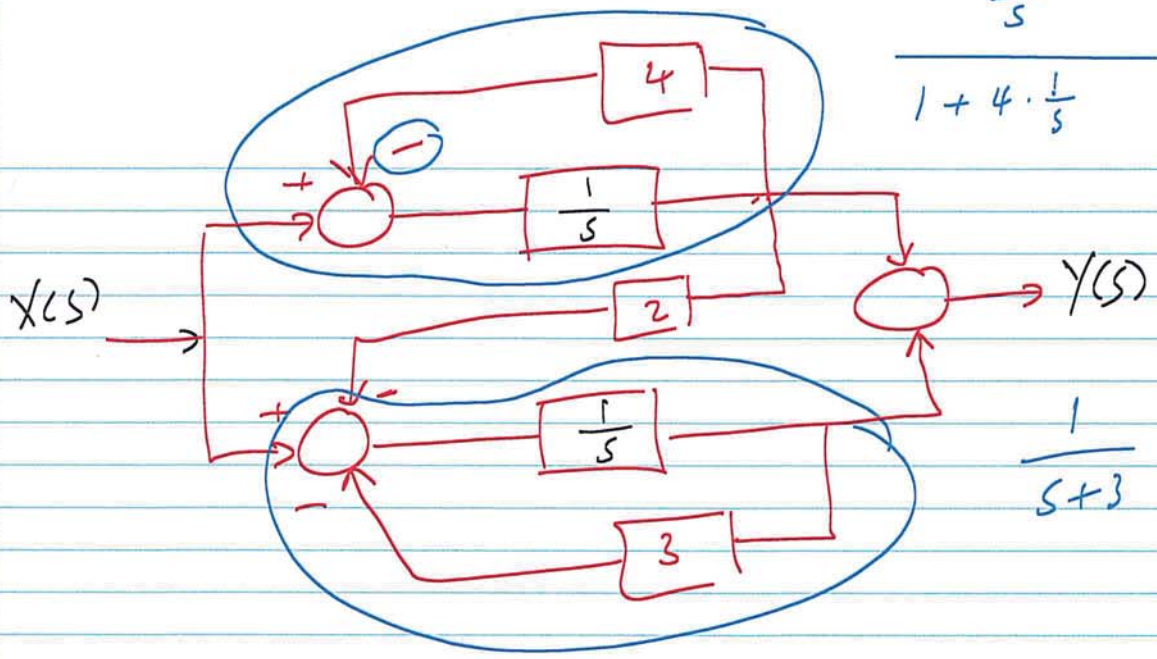
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$$H_1(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)}{1+H_1(s)H_2(s)}$$

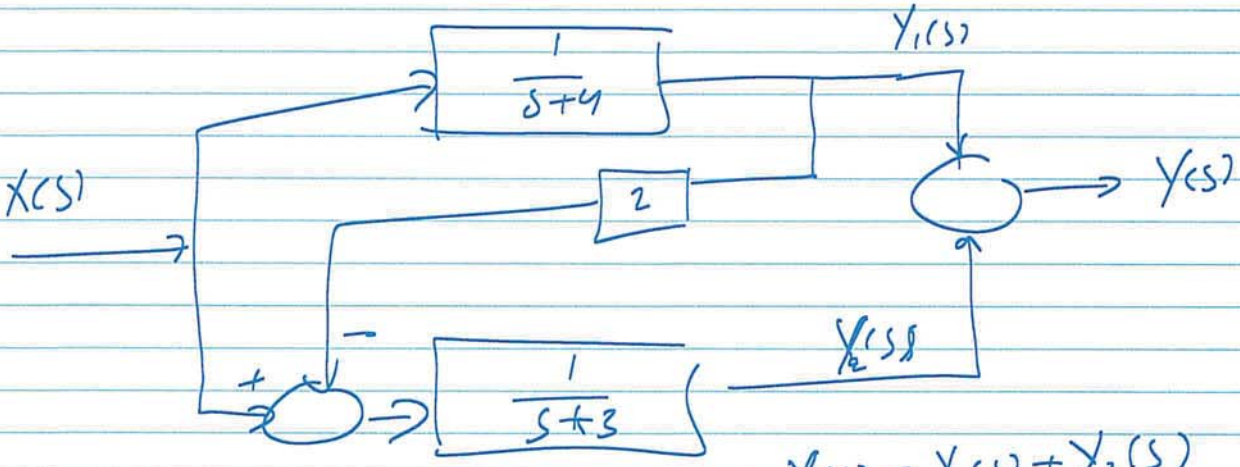
$$\frac{Y(s)}{X(s)} = H_1(s) + H_2(s)$$

$$H_1(s) = \frac{Y(s)}{X(s)} = H_1(s)H_2(s)$$

$$\frac{\frac{1}{s}}{1+4\cdot\frac{1}{s}} = \frac{1}{s+4}$$



$$\frac{1}{s+3}$$



$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_1(s) = X(s)\left(\frac{1}{s+4}\right)$$

$$Y_2(s) = \left(X(s) - 2Y_1(s)\right)\left(\frac{1}{s+3}\right)$$

$$Y_2(s) = \left(X(s) - 2X(s)\left(\frac{1}{s+4}\right)\right)\left(\frac{1}{s+3}\right)$$

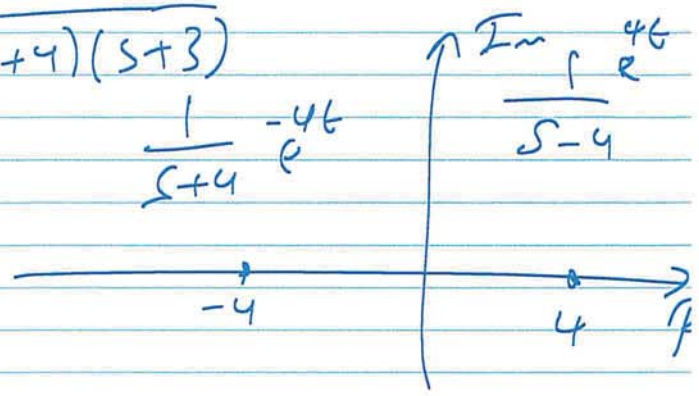
$$Y(s) = X(s)\left(\frac{1}{s+4}\right) + X(s)\left(\frac{1}{s+3}\right) - 2X(s)\left(\frac{1}{s+4}\right)\left(\frac{1}{s+3}\right)$$

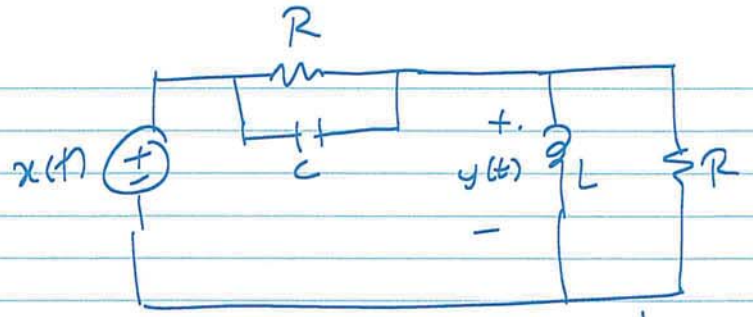
$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{s+4} + \frac{1}{s+3} - 2\left(\frac{1}{s+4}\right)\left(\frac{1}{s+3}\right)$$

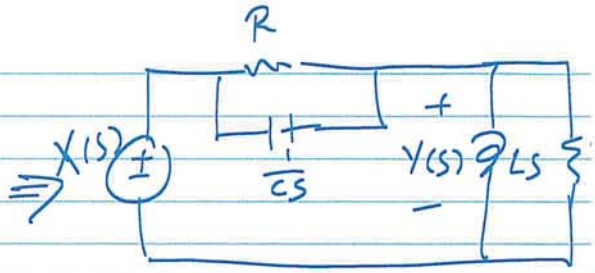
$$H(s) = \frac{1}{s+4} + \frac{1}{s+3} - \frac{2}{(s+4)(s+3)}$$

$+4t$
$e^{\quad}$

$\frac{1}{s+4}$ e^{-4t}

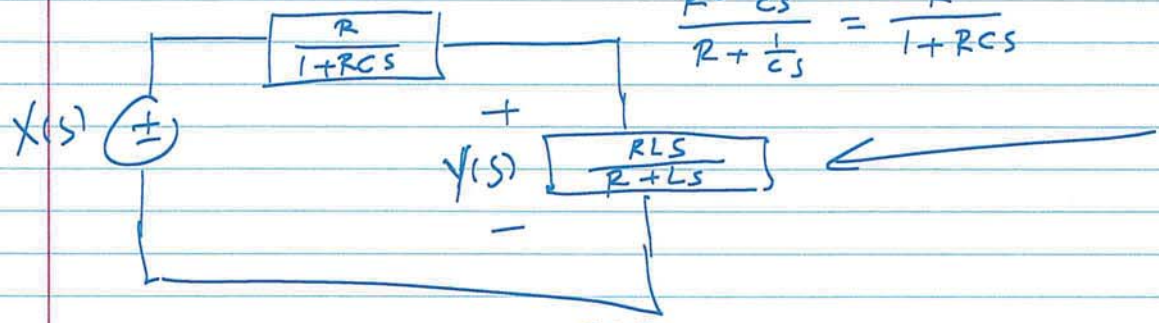




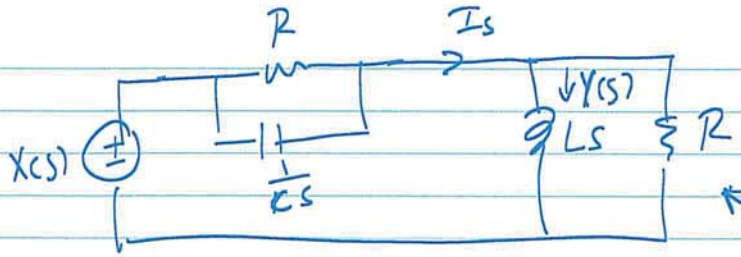


$$\frac{R \cdot \frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{1+RCs}$$

$$\frac{R \cdot sL}{R + sL}$$

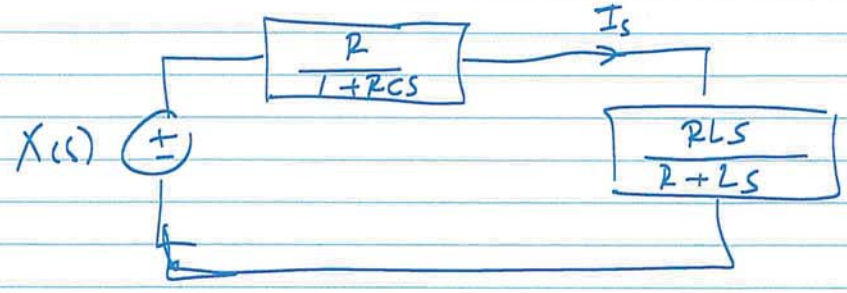


$$Y(s) = X(s) \frac{\frac{RLs}{R+Ls}}{\frac{RLs}{R+Ls} + \frac{R}{1+RCs}}$$



$$Y(s) = I_s\left(\frac{R}{R+Ls}\right)$$

$$= \frac{X(s)}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}\left(\frac{R}{R+Ls}\right)$$



$$\frac{Y(s)}{X(s)} = \frac{\frac{R}{R+Ls}}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}$$

$$I_s = \frac{X(s)}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}$$

* $X(s) = \dfrac{10}{s^2+7s+10}$, find $x(t)$

$$X(s) = \frac{10}{(s+2)(s+5)} = \frac{C_1}{s+2} + \frac{C_2}{s+5}$$

$$C_1 = (s+2)X(s)\Big|_{s=-2} = \frac{10}{s+5}\Big|_{s=-2} = \frac{10}{3}$$

$$C_2 = (s+5)X(s)\Big|_{s=-5} = \frac{10}{s+2}\Big|_{s=-5} = \frac{10}{-3} = -\frac{10}{3}$$

$$X(s) = \frac{10/3}{s+2} - \frac{5}{s+5}$$

$$x(t) = \frac{10}{3}e^{-2t} - \frac{10}{3}e^{-5t}, \quad t \geq 0$$

$$X(s) = \frac{10}{(s+2)^2(s+5)} = \frac{c_1}{s+2} + \frac{c_2}{(s+2)^2} + \frac{c_3}{s+5}$$

$$c_2 = (s+5)X(s)\Big|_{s=-5} = \frac{10}{(s+2)^2}\Big|_{s=-5} = \frac{10}{(-3)^2} = \frac{10}{9}$$

$$c_2 = (s+2)^2 X(s)\Big|_{s=-2} = \frac{10}{s+5}\Big|_{s=-2} = \frac{10}{3}$$

$$\frac{10}{(s+2)^2(s+5)} = \frac{c_1(s+4)(s+5) + c_2(s+5) + c_3(s+2)^2}{(s+2)^2(s+5)}$$

$$10 = c_1(s^2+7s+10) + c_2 s + c_2 5 + c_3(s^2+4s+4)$$

$$10 = (c_1+c_3)s^2 + (7c_1 + c_2 + 4c_3)s + (10c_1 + 5c_2 + 4c_3)$$

$$c_1 + c_3 = 0 \Rightarrow c_1 = -\frac{10}{9}$$

$$7c_1 + c_2 + 4c_3 = 0 \qquad 10c_1 + 5c_2 + 4c_3 = 10$$

$$c_{r-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}\left[(s-p_1)^r X(s)\right]\right]_{s=p_1}$$

$r = 2$

$$c_1 = \frac{1}{1!}\left[\frac{d}{ds}\left[(s+2)^2 X(s)\right]\right]\Big|_{s=-2}$$

$$c_1 = \frac{d}{ds}\left[\frac{10}{s+5}\right]\Big|_{s=-2}$$

$$= \frac{d}{ds}\left[10(s+5)^{-1}\right]\Big|_{s=-2}$$

$$= -10(s+5)^{-2}\Big|_{s=-2}$$

$$= \frac{-10}{(s+5)^2}\Big|_{s=-2} = \frac{-10}{(-2+5)^2} = \frac{-10}{3^2} = \frac{-10}{9}$$

$$C_1 = \frac{-10}{9}$$