Note Title 4/24/2018

$$X(s) = \frac{s+2}{s^3+6s^2+9s} = \frac{s+2}{s(s+3)^2}$$

$$X(s) = \frac{C_1}{s} + \frac{C_2}{s+3} + \frac{C_3}{(s+3)^2}$$

$$C_1 = sX(s)\Big|_{s=0} = \frac{s+2}{(s+3)^2}\Big|_{s=0} = \frac{2}{9}$$

$$C_3 = (s+3)^2 X(s)\Big|_{s=-3} = \frac{s+2}{s}\Big|_{s=-3} = \frac{1}{3}$$

Find C_2

(1) $$C_2 = \frac{d}{ds}\left[(s+3)^2 X(s)\right]\Big|_{s=-3} = \frac{d}{ds}\left[\frac{s+2}{s}\right]\Big|_{s=-3}$$

$$= \frac{s-(s+2)}{s^2}\Big|_{s=-3} = \frac{-2}{s^2}\Big|_{s=-3} = \frac{-2}{9}$$

(2)

$$X(s) = \frac{C_1}{s} + \frac{C_2}{s+3} + \frac{C_3}{(s+3)^2}$$

$$X(s) = \frac{2/9}{s} + \frac{C_2}{s+3} + \frac{1/3}{(s+3)^2} = \frac{s+2}{s(s+3)^2}$$

$$s(s+3)^2\left[\frac{2/9}{s} + \frac{C_2}{s+3} + \frac{1/3}{(s+3)^2}\right] = s+2$$

$$\frac{2}{9}(s+3)^2 + C_2(s(s+3)) + \frac{1}{3}s = s+2$$

$$\frac{2}{9}(s^2+6s+9) + C_2(s^2+3s) + \frac{1}{3}s = s+2$$

$$s^2\left(\frac{2}{9}+C_2\right) + s\left(\frac{6\times 2}{9} + 3C_2 + \frac{1}{3}\right) + \frac{2\times 9}{9} = s+2$$

$$\frac{2}{9} + C_2 = 0 \Rightarrow C_2 = -\frac{2}{9}$$

or $$\frac{12}{9} + 3C_2 + \frac{1}{3} = 1 \Rightarrow C_2 = -\frac{2}{9}$$

$$X(s) = \frac{s+4}{s^2+4s+4} = \frac{s+4}{(s+2)^2} = \frac{C_1}{(s+2)} + \frac{C_2}{(s+2)^2}$$

$$C_2 = (s+2)^2 X(s)\Big|_{s=-2} = (s+2)^2\left(\frac{s+4}{(s+2)^2}\right)\Big|_{s=-2} = (s+4)\Big|_{s=-2} = 2$$

C_1

① $$C_1 = \frac{d}{ds}\left[(s+2)^2 X(s)\right]\Big|_{s=-2} = \frac{d}{ds}(s+4)\Big|_{s=-2} = 1$$

OR

② $$\frac{C_1}{s+2} + \frac{C_2}{(s+2)^2} = \frac{s+4}{(s+2)^2}$$

$$\therefore \frac{C_1}{s+2} + \frac{2}{(s+2)^2} = \frac{s+4}{(s+2)^2}$$

$$C_1(s+2) + 2 = s+4$$

$$C_1 s + 2C_1 + 2 = s+4$$

$$C_1 s = s \Rightarrow C_1 = 1$$

or $$2C_1 + 2 = 4 \Rightarrow C_1 = 1$$

$$X(s) = \frac{1}{s+2} + \frac{2}{(s+2)^2}$$

$$t^N e^{-bt} u(t) \longleftrightarrow \frac{N!}{(s+b)^{N+1}}$$

$\therefore N = 1$

$$2t e^{-2t} \longleftrightarrow \frac{2}{(s+2)^2}$$

$$\therefore x(t) = \left(e^{-2t} + 2t e^{-2t}\right) u(t) = (1+2t) e^{-2t} u(t)$$

$$\frac{dy(t)}{dt} + a\,y(t) = b\,x(t)$$

$$S\,Y(s) - y(0) + a\,Y(s) = b\,X(s)$$

$$Y(s)\,[S + a] = y(0) + b\,X(s)$$

$$\therefore\ Y(s) = \frac{y(0)}{s+a} + \frac{b\,X(s)}{s+a}$$

(1) If $y(0) = 2$, $a = 3$, and $b = 4$, $x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$$Y(s) = \frac{2}{s+3} + \frac{4}{s+3}\left(\frac{1}{s}\right) = \frac{2}{s+3} + \frac{4}{s(s+3)}$$

$$= \frac{2}{s+3} + \frac{A}{s} + \frac{B}{s+3}$$

$$y(t) = 2e^{-3t} + A + Be^{-3t}, \quad t \geq 0 \text{ or } u(t)$$

(2) If $y(0) = 0$ or I.C. $= 0$

$$\therefore\ Y(s) = \frac{b\,X(s)}{s+a} \Rightarrow \boxed{\frac{Y(s)}{X(s)} = \frac{b}{s+a}}$$

↑ Transfer function

To find T.F., assume I.C. $= 0$

$$\frac{d^2y(t)}{dt^2} + a\,\frac{dy(t)}{dt} + b\,y(t) = c\,\frac{dx(t)}{dt} + d\,x(t)$$

$$S^2Y(s) - s\,y(0) - \dot{y}(0) + a\,s\,Y(s) - y(0) + b\,Y(s) = c\,s\,X(s) - x(0) + d\,X(s)$$

If I.C. $= 0$

$$s^2 Y(s) + a s Y(s) + b Y(s) = c s X(s) + d X(s)$$

$$Y(s)\left(s^2 + as + b\right) = X(s)\left(cs + d\right)$$

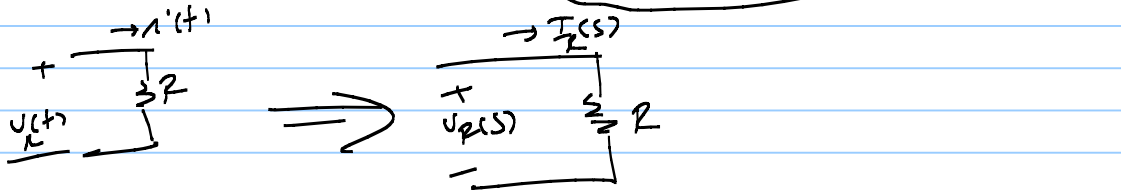
$$\boxed{\frac{Y(s)}{X(s)} = \frac{cs + d}{s^2 + as + b}}$$

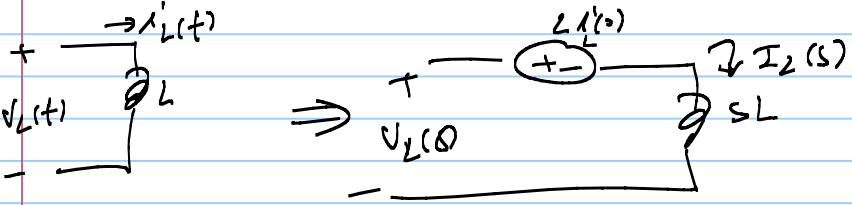
$$v_R(t) = R\,i'(t) \quad \Rightarrow \quad V_R(s) = R\,I(s)$$

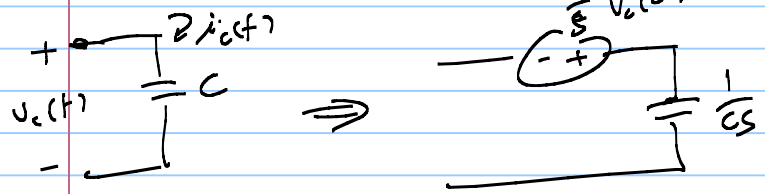
$$v_L(t) = L\frac{di'(t)}{dt} \quad \Rightarrow \quad V_L(s) = Ls\,I(s) - L\,i'(0)$$

$$i_c(t) = c\frac{dv_c(t)}{dt} \Rightarrow I_c(s) = cs\,V(s) - c\,v_c(0)$$

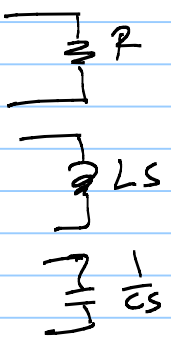
$$\boxed{V(s) = \frac{1}{cs} I_c(s) + \frac{1}{s} v_c(0)}$$



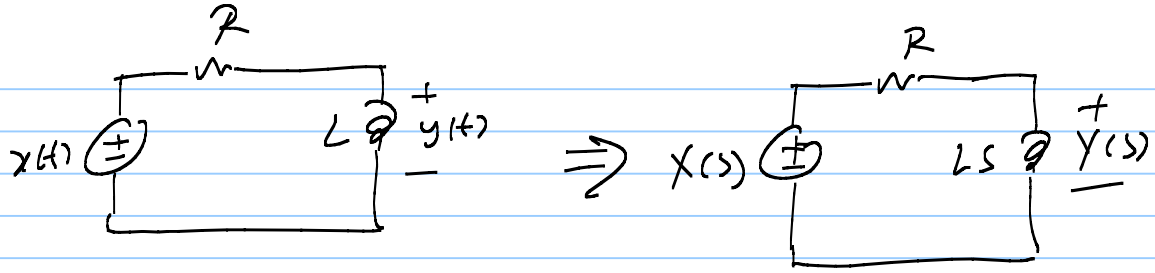




If $I.C. = 0$



Ex



find the T.F.

$$Y(s) = X(s)\left(\frac{Ls}{R+Ls}\right)$$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{Ls}{R+Ls} = \frac{s}{s+R/L}$$