(1) Determine the Laplace transform of the following signals

(a) $x(t) = e^{-5t}\cos(4t)\,u(t)$

Let $x_1(t) = \cos(4t)\,u(t) \Rightarrow X_1(s) = \dfrac{s}{s^2+4^2}$

$$\Rightarrow X(s) = \frac{s+5}{(s+5)^2+4^2}$$

(b) $x(t) = (4 - 2e^{-5t})\,u(t) \Rightarrow x(t) = 4u(t) - 2e^{-5t}u(t)$

$$\Rightarrow X(s) = \frac{4}{s} - \frac{2}{s+5}$$

(c) $x(t) = e^{-10t}\cos(3t-1)\,u(t)$

$$\cos(3t-1) = \cos(3t)\cos(1) + \sin(3t)\sin(1)$$

$$\therefore x(t) = e^{-10t}\cos(3t)\cos(1) + e^{-10t}\sin(3t)\sin(1)$$

$$\therefore X(s) = \frac{\cos(1)(s+10)}{(s+10)^2+3^2} + \frac{\sin(1)(3)}{(s+10)^2+3^2}$$

(2) A continuous-time signal $x(t)$ has the Laplace transform

$$X(s) = \frac{s+1}{s^2+5s+7}$$

Determine the Laplace transform $V(s)$ of the following signals:

(a) $v(t) = x(3t-4)\,u(3t-4)$

$$x(3t)\,u(3t) \longleftrightarrow \frac{1}{3}X\left(\frac{s}{3}\right)$$

The time shift of this signal is $3t = 4 \Rightarrow t = \frac{4}{3}$

$$\therefore V(s) = \frac{1}{3}X\left(\frac{s}{3}\right)e^{-\frac{4}{3}s}$$

$$\therefore V(s) = \frac{1}{3}\,\frac{\frac{s}{3}+1}{\left(\frac{s}{3}\right)^2+5\left(\frac{s}{3}\right)+7}\,e^{-\frac{4}{3}s}$$

$$= \frac{\frac{s}{9}+\frac{1}{3}}{\frac{s^2}{9}+\frac{5s}{3}+7}\,e^{-\frac{4}{3}s} = \frac{s+3}{s^2+15s+63}\,e^{-\frac{4}{3}s}$$

(b) $v(t) = t\,x(t)$

$$V(s) = -\frac{d}{ds}\left(\frac{s+1}{s^2+5s+7}\right) = -\frac{(s^2+5s+7)-(s+1)(2s+5)}{(s^2+5s+7)^2}$$

$$= -\frac{s^2+5s+7-2s^2-7s-5}{(s^2+5s+7)^2} = \frac{s^2+2s-2}{(s^2+5s+7)^2}$$

(c) $v(t) = x(t) * x(t)$

$$V(s) = X^2(s) = \frac{(s+1)^2}{(s^2+5s+7)^2} = \frac{s^2+2s+1}{(s^2+5s+7)^2}$$

(d) $v(t) = e^{-3t}\,x(t)$

$$V(s) = X(s+3) = \frac{(s+3)+1}{(s+3)^2+5(s+3)+7} = \frac{s+4}{s^2+11s+31}$$

(e) $v(t) = x(t)\sin 2t$

$$V(s) = \frac{j}{2}\left[X(s+j2) - X(s-j2)\right]$$

$$= \frac{j}{2}\left[\frac{s+j2+1}{(s+j2)^2+5(s+j2)+7} - \frac{s-j2+1}{(s-j2)^2+5(s-j2)+7}\right]$$

$$= \frac{j}{2}\left[\frac{s+j2+1}{s^2+j4s-4+5s+j10+7} - \frac{s-j2+1}{s^2-j4s-4+5s-j10+7}\right]$$

$$= \frac{2s^2+4s+4}{s^4+10s^3+47s^2+110s+109}$$

(f) $v(t) = \int_0^t x(\tau)\,d\tau$

$$V(s) = \frac{1}{s} X(s) = \frac{s+1}{s(s^2+5s+7)}$$

(g) $v(t) = \frac{d^2x(t)}{dt^2}$

$$V(s) = s^2X(s) - s\,X(0) - \dot{X}(0)$$

$$X(0) = \lim_{s\to\infty} s\,X(s) = \lim_{s\to\infty} \frac{s(s+1)}{s^2+5s+7} = \lim_{s\to\infty} \frac{1+\frac{1}{s^2}}{1+\frac{5}{s}+\frac{7}{s^2}} = 1$$

$$\dot{X}(0) = \lim_{s\to\infty} \left[s^2X(s) - s\,X(0)\right] = \lim_{s\to\infty} \left[\frac{s^3+s^2}{s^2+5s+7} - s(1)\right]$$

$$= \lim_{s\to\infty} \left[\frac{s^3+s^2-s(s^2+5s+7)}{s^2+5s+7}\right]$$

$$= \lim_{s\to\infty} \left[\frac{-4s^2-7s}{s^2+5s+7}\right]$$

$$= \lim_{s\to\infty} \left[\frac{-4-\frac{7}{s}}{1+\frac{5}{s}+\frac{7}{s^2}}\right] = -4$$

$$\therefore V(s) = \frac{s^2(s+1)}{s^2+5s+7} - s + 4 = \frac{13s+28}{s^2+5s+7}$$

(3) Determine the final values $[\lim_{t\to\infty} x(t)]$ of the following signals

a) $X(s) = \frac{4}{s^2+s}$

$$x(\infty) = \lim_{s\to 0} s\,X(s) = \lim_{s\to 0} \frac{4}{s+1} = 4$$

(b) $X(s) = \frac{3s+4}{s^2+s}$

$$x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{3s+4}{s+1} = 4$$

(c) $X(s) = \frac{4}{s^2-s}$, poles at $s = 0, +1$

Since it has a pole at $s = 1$, then $x(\infty)$ is not finite.

(4) Determine the initial values $x(0)$ of the following signals

(a) $X(s) = \frac{4}{s^2+s}$

$$x(0) = \lim_{s\to\infty} s\,X(s) = \lim_{s\to\infty} \frac{4}{s+1} = 0$$

(b) $X(s) = \frac{3s+4}{s^2+s}$

$$x(0) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{3s+4}{s+1} = \lim_{s\to\infty} \frac{3+\frac{4}{s}}{1+\frac{1}{s}} = 3$$

(c) $X(s) = \frac{4}{s^2-s}$

$$x(0) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{4}{s-1} = 0$$

① Determine the inverse Laplace transform of the following functions. Use MATLAB to verify your answer.

(a) $X(s) = \dfrac{s+2}{s^2+7s+12} \Rightarrow X(s) = \dfrac{s+2}{(s+4)(s+3)} = \dfrac{c_1}{s+4} + \dfrac{c_2}{s+3}$

$$c_1 = (s+4)X(s)\Big|_{s=-4} = (s+4)\frac{s+2}{(s+4)(s+3)}\Big|_{s=-4} = 2$$

$$c_2 = (s+3)X(s)\Big|_{s=-3} = (s+3)\frac{s+2}{(s+4)(s+3)}\Big|_{s=-3} = -1$$

$$\therefore X(s) = \frac{2}{s+4} - \frac{1}{s+3}$$

from Table 6.2 $\quad e^{-bt}u(t) \Longleftrightarrow \dfrac{1}{s+b}$

$$\therefore x(t) = 2e^{-4t} - e^{-3t}, \quad t \geq 0$$

(b) $X(s) = \dfrac{s+1}{s^3+5s^2+7s} \Rightarrow$ (use MATLAB "roots") $X(s) = \dfrac{s+1}{s(s+2.5-j0.866)(s+2.5+j0.866)}$

$$\therefore X(s) = \frac{c_1}{s} + \frac{c_2}{(s+2.5-j0.866)} + \frac{\overline{c_2}}{s+2.5+j0.866}$$

$$c_1 = [sX(s)]\Big|_{s=0} = \frac{s+1}{s^2+5s+7}\Big|_{s=0} = \frac{1}{7}$$

$$c_2 = [(s+2.5-j0.866)X(s)]\Big|_{s=-2.5+j0.866} = \frac{s+1}{s(s+2.5+j0.866)}\Big|_{s=-2.5+j0.866}$$

$$\therefore c_2 = \frac{-1.5+j0.866}{(-2.5+j0.866)(j2*0.866)} = 0.3780\angle -100.89^\circ$$

$|C_2| = 0.3780$, $\angle C_2 = -100.89^\circ$

$x(t) = C_1 e^{p_1 t} + 2|C_2| e^{\sigma t} \cos(\omega t + \angle C_2)$

$\sigma + j\omega = -2.5 + j0.866$

$\therefore x(t) = \frac{1}{7} e^{0(t)} + 2 * 0.3780 * e^{-2.5t} \cos(0.866t - 100.89^\circ)$

$\therefore x(t) = \frac{1}{7} + 0.7559\, e^{-2.5t} \cos(0.866t - 100.89^\circ)$, $t \geq 0$

(c) $Y(s) = \frac{2s^2 - 9s - 35}{s^2 + 4s + 2}$ $\quad M = N = 2$

$X(s) = Q(s) + \frac{R(s)}{A(s)}$, use deconv([2 -9 -35], [1 4 2])

gives $Q = 2$ & $R = 0, -17, -39$

$\therefore X(s) = 2 + \frac{-17s - 39}{s^2 + 4s + 2}$

Let $X_1(s) = \frac{-17s - 39}{s^2 + 4s + 2}$

Use roots([1 4 2]) gives -3.4142 & -0.5858

Use
[r, p] = residue([-17 -39], [1 4 2]) gives $r = -6.7322$ & -10.2678

$\therefore X(s) = 2 - \frac{6.7322}{(s + 3.4142)} - \frac{10.2678}{(s + 0.5858)}$

$\therefore x(t) = 2\delta(t) - 6.7322\, e^{-3.4142t} - 10.2678\, e^{-0.5858t}$, $t \geq 0$

(d) $X(s) = \dfrac{3s^2+2s+1}{s^3+5s^2+8s+4}$ use roots([1 5 8 4]) gives $-2, -2, -1$

$$\therefore X(s) = \frac{C_1}{(s+2)} + \frac{C_2}{(s+2)^2} + \frac{C_3}{(s+1)} = \frac{3s^2+2s+1}{(s+2)^2(s+1)}$$

$$C_3 = [(s+1)X(s)]_{s=-1} = \left.\frac{3s^2+2s+1}{(s+2)^2}\right|_{s=-1} = \frac{2}{1} = 2$$

$$C_2 = [(s+2)^2 X(s)]_{s=-2} = \left.\frac{3s^2+2s+1}{s+1}\right|_{s=-2} = \frac{9}{-1} = -9$$

$$C_1 = \left[\frac{d}{ds}[(s+2)^2 X(s)]\right]_{s=-2} = \left[\frac{(s+1)(6s+2) - (3s^2+2s+1)(1)}{(s+1)^2}\right]\Bigg|_{s=-2}$$

$$= \frac{(-1)(-12+2) - (12-4+1)}{1} = 1$$

$$\therefore X(s) = \frac{1}{(s+2)} - \frac{9}{(s+2)^2} + \frac{2}{(s+1)}$$

$$\therefore x(t) = e^{-2t} - 9te^{-2t} + 2e^{-t}, \quad t \geq 0$$

(e) $X(s) = \dfrac{s+2}{s^2+10s+25} \Rightarrow X(s) = \dfrac{s+2}{(s+5)^2} = \dfrac{C_1}{(s+5)} + \dfrac{C_2}{(s+5)^2}$

$$C_2 = [(s+5)^2 X(s)]_{s=-5} = \left.\frac{s+2}{1}\right|_{s=-5} = -3$$

$$C_1 = \left[\frac{d}{ds}[(s+1)^2 X(s)]\right]_{s=-5} = \left.\frac{d}{ds}(s+2)\right|_{s=-5} = 1$$

$$\therefore X(s) = \frac{1}{s+5} - \frac{3}{(s+5)^2}$$

$$\therefore x(t) = e^{-5t} - 3te^{-5t}$$

MATLAB

(a)

```
syms X s x
X=(s+2)/(s^2+7*s+12);
x=ilaplace(X)
```

x =2*exp(-4*t) - exp(-3*t)

(b)

```
syms X s x
X=(s+1)/(s^3+5*s^2+7*s);
x=ilaplace(X)
```

x =1/7 - (exp(-(5*t)/2)*(cos((3^(1/2)*t)/2) - 3*3^(1/2)*sin((3^(1/2)*t)/2)))/7

(c)

```
syms X s x
X=(2*s^2-9*s-35)/(s^2+4*s+2);
x=ilaplace(X)
```

x =2*dirac(t) - 17*exp(-2*t)*(cosh(2^(1/2)*t) + (5*2^(1/2)*sinh(2^(1/2)*t))/34)

(d)

```
syms X s x
X=(3*s^2+2*s+1)/(s^3+5*s^2+8*s+4);
x=ilaplace(X)
```

x =2*exp(-t) + exp(-2*t) - 9*t*exp(-2*t)

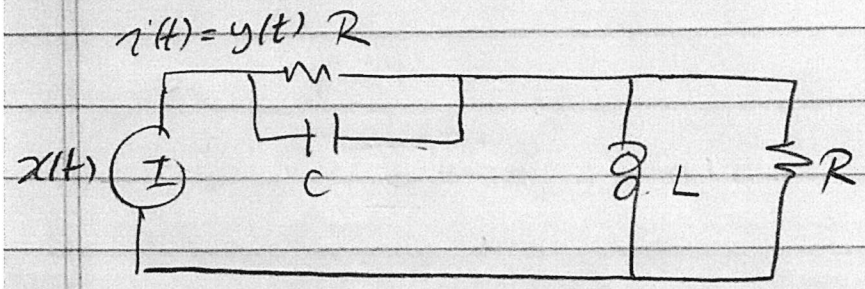
(e)

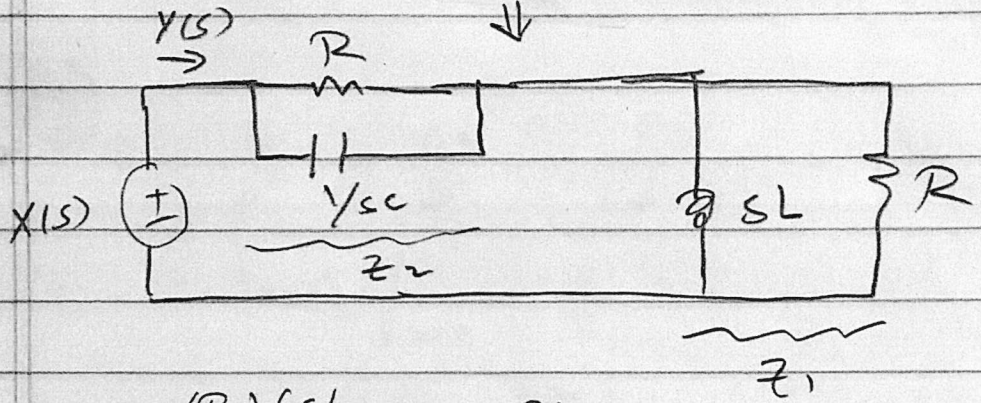
```
syms X s x
X=(s+2)/(s^2+10*s+25);
x=ilaplace(X)
```

x =exp(-5*t) - 3*t*exp(-5*t)

Laplace

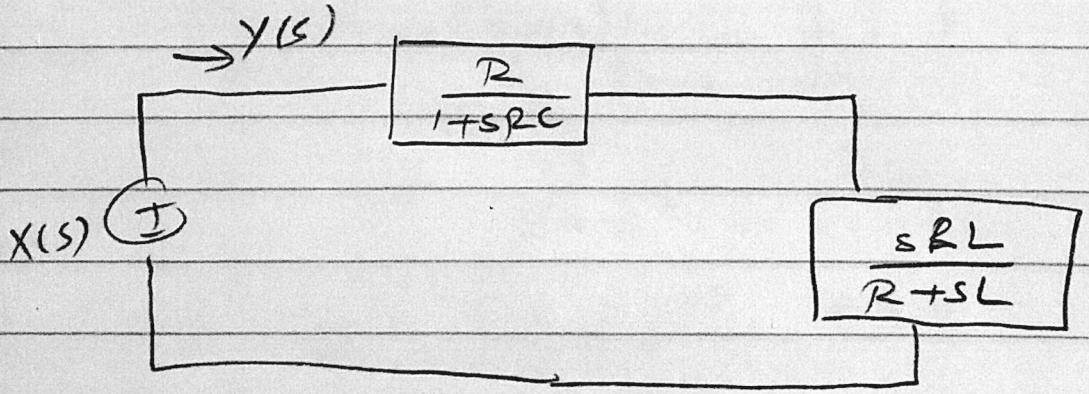
① Find the transfer function of the following system





$$Z_1 = \frac{(R)(sL)}{R+sL} = \frac{sRL}{R+sL}$$

$$Z_2 = \frac{(R)(1/sC)}{R+1/sC} = \frac{R}{1+sRC}$$



$$\frac{Y(s)}{X(s)} = \frac{1}{\frac{R}{1+sRC} + \frac{sRL}{R+sL}} = \frac{(1+sRC)(R+sL)}{R(R+sL) + sRL(1+sRC)}$$

$$H(s) = \frac{R + s(R^2C + L) + s^2RLC}{R^2 + 2sRL + s^2R^2LC}$$

(2) Find the inverse Laplace transform

$$X(s) = \frac{s+1}{s^4+4s^3+4s^2}$$

$$X(s) = \frac{s+1}{s^2(s^2+4s+4)} = \frac{s+1}{s^2(s+2)^2}$$

$$X(s) = \frac{c_1}{s^2} + \frac{c_2}{s} + \frac{c_3}{(s+2)^2} + \frac{c_4}{(s+2)}$$

$$c_1 = [s^2 X(s)]\Big|_{s=0} = \frac{1}{4}$$

$$c_2 = \left[\frac{d}{ds}(s^2 X(s))\right]\Big|_{s=0} = \frac{(s+2)^2 - (s+1)(2(s+2))}{(s+2)^4}\Big|_{s=0} = 0$$

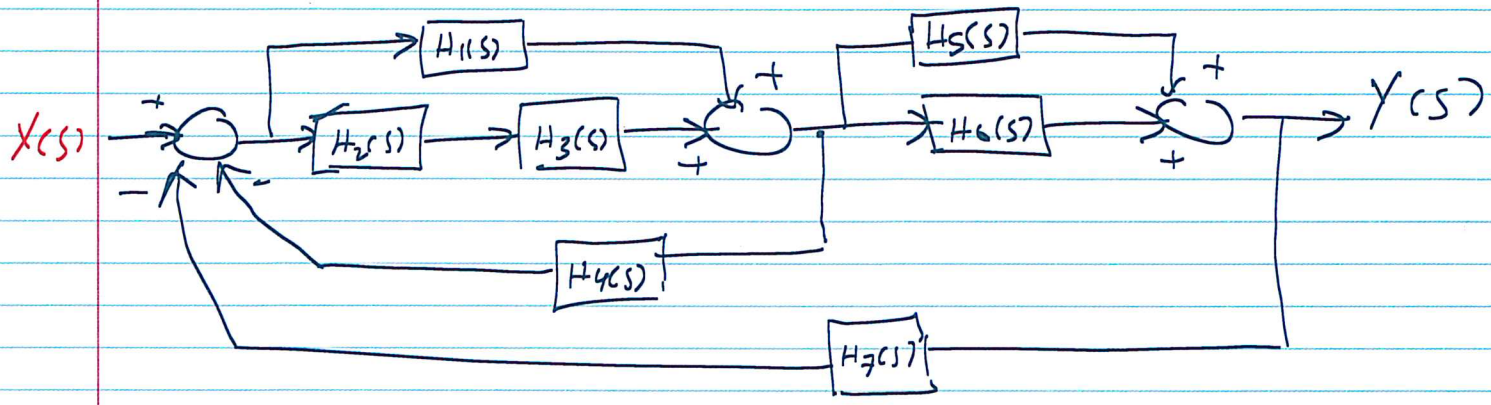
$$c_3 = [(s+2)^2 X(s)]\Big|_{s=-2} = \frac{s+1}{s^2}\Big|_{s=-2} = -\frac{1}{4}$$

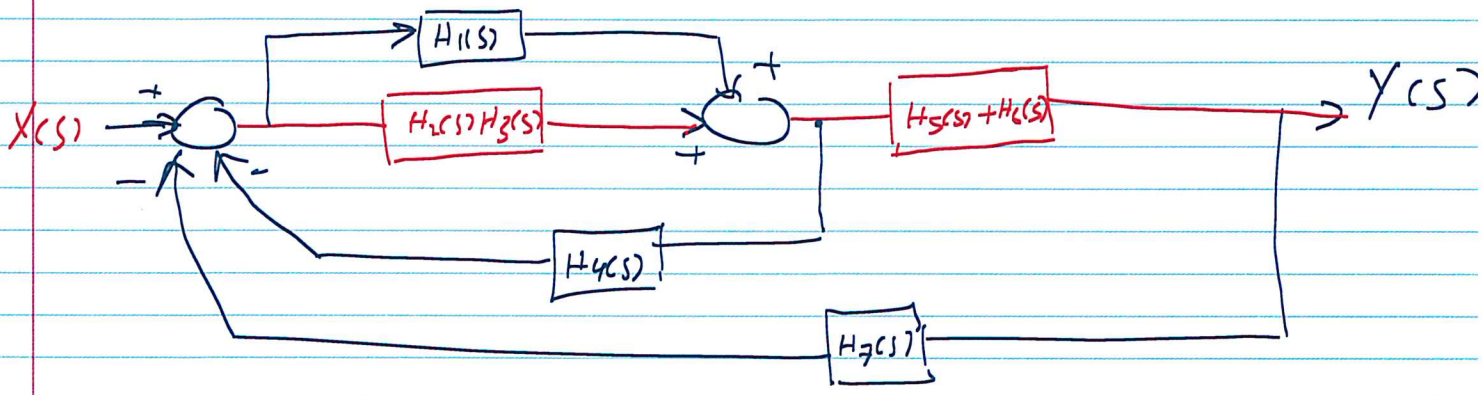
$$c_4 = \left[\frac{d}{ds}((s+2)^2 X(s))\right]_{s=-2} = \frac{d}{ds}\left(\frac{s+1}{s^2}\right)\Big|_{s=-2} = 0$$

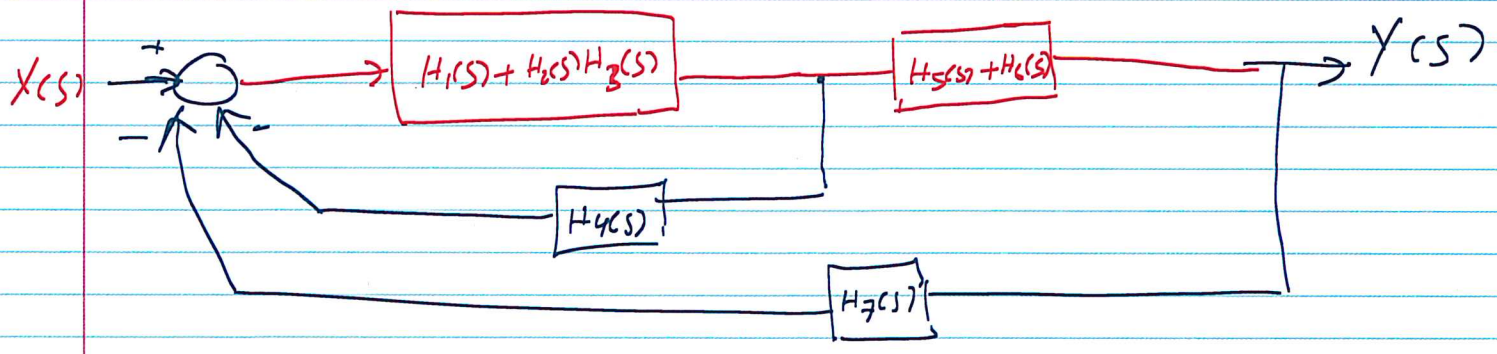
$$\Rightarrow X(s) = \frac{1/4}{s^2} - \frac{1/4}{(s+2)^2}$$

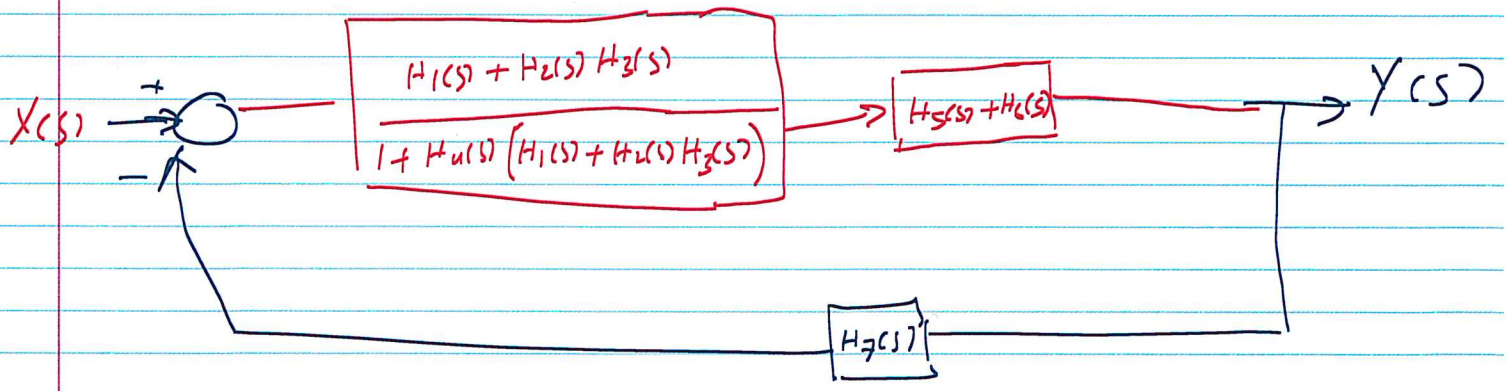
$$\therefore x(t) = \frac{1}{4}t - \frac{1}{4}te^{-2t}, \quad t \geq 0$$

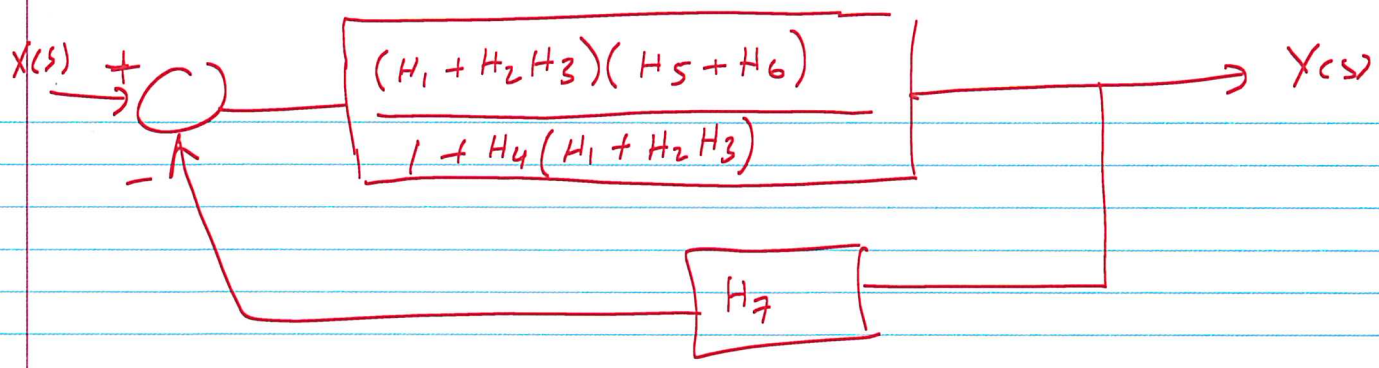
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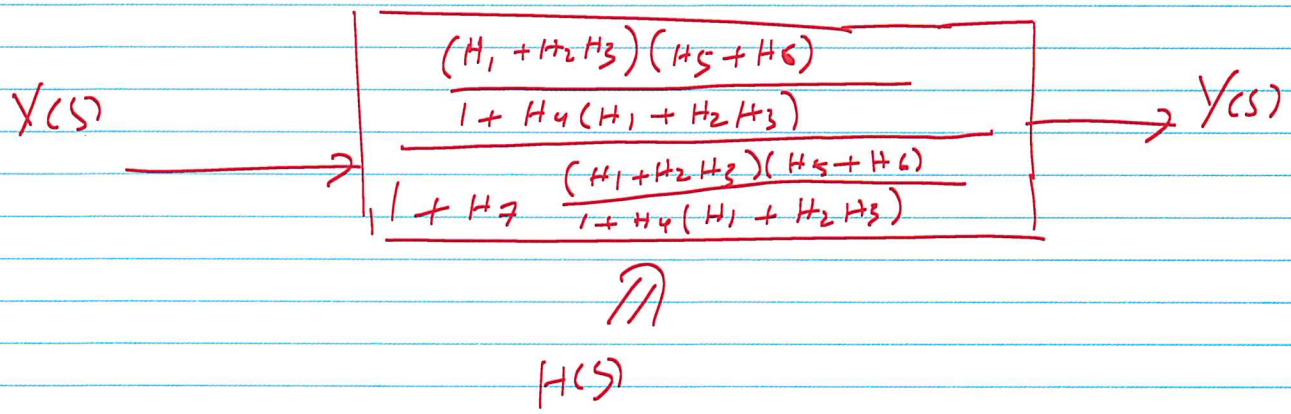




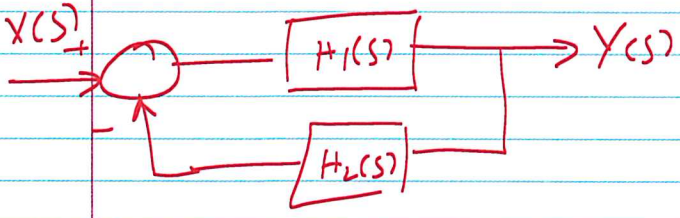




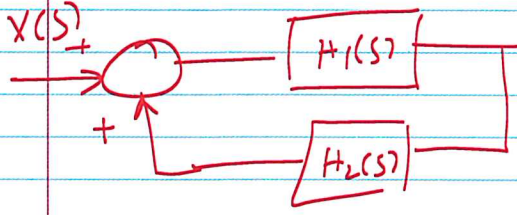




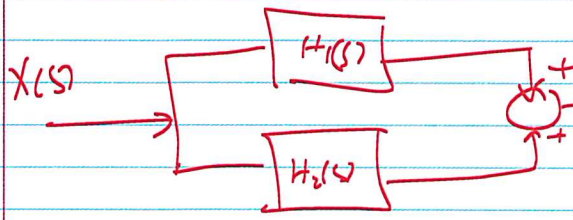
$$\Rightarrow H(s)$$



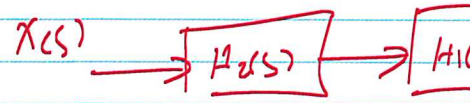
$$\Rightarrow H(s) = \frac{H_1(s)}{1 + H_2(s)\,H_1(s)}$$



$$\Rightarrow H(s) = \frac{H_1(s)}{1 - H_2(s)\,H_1(s)}$$

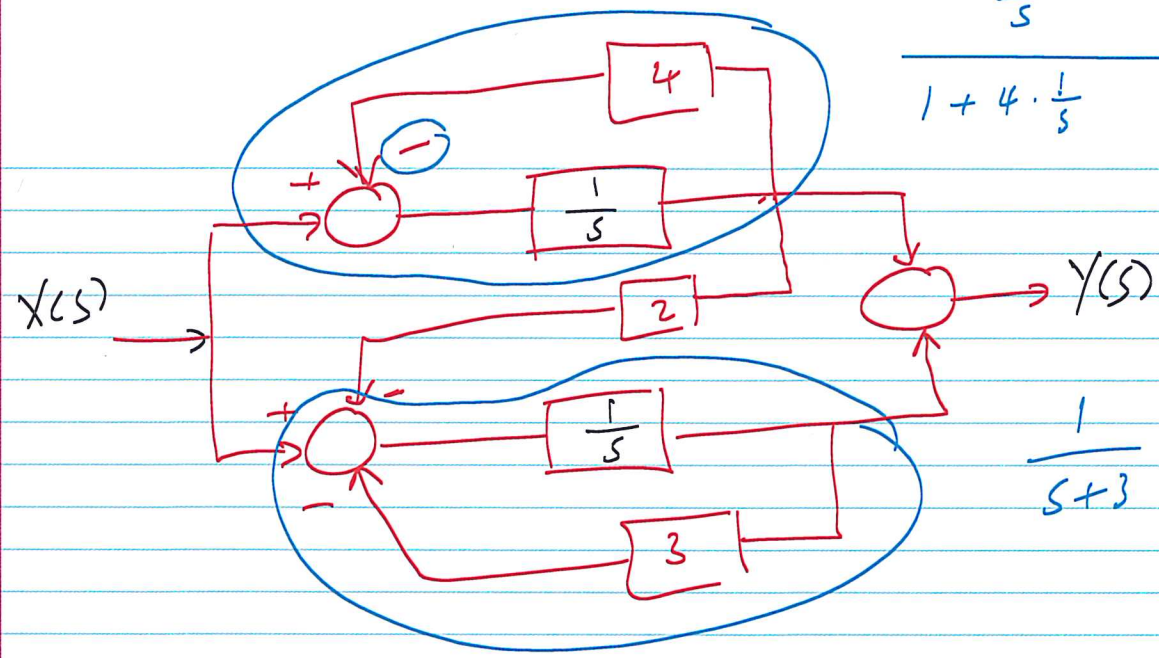


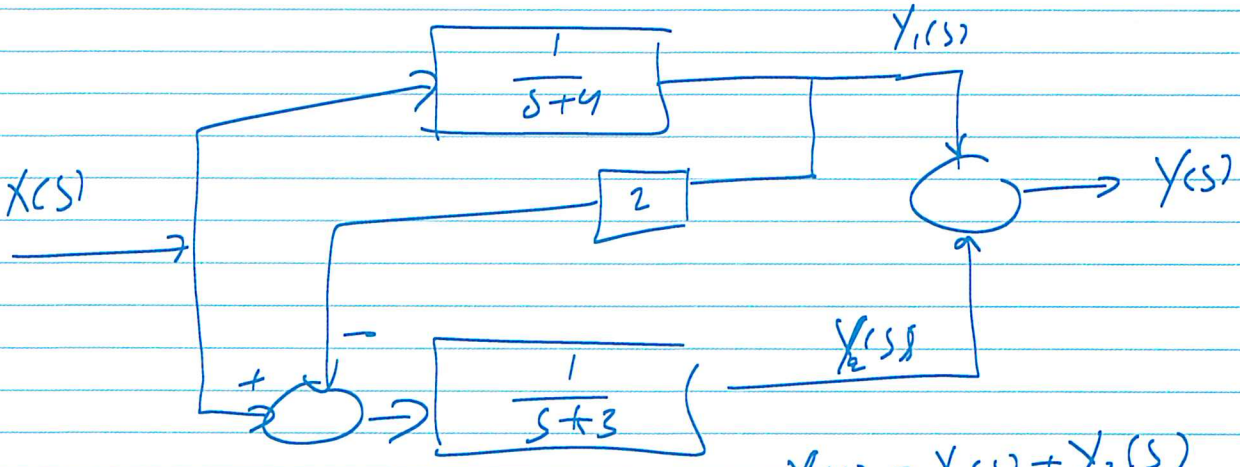
$$\Rightarrow H(s) = H_1(s) + H_2(s)$$



$$\Rightarrow H(s) = H_1(s)\,H_2(s)$$

$$\frac{\frac{1}{s}}{1+4\cdot\frac{1}{s}} = \frac{1}{s+4}$$





$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_1(s) = X(s)\left(\frac{1}{s+4}\right)$$

$$Y_2(s) = \left(X(s) - 2Y_1(s)\right)\left(\frac{1}{s+3}\right)$$

$$Y_2(s) = \left(X(s) - 2X(s)\left(\frac{1}{s+4}\right)\right)\left(\frac{1}{s+3}\right)$$

$$Y(s) = X(s)\left(\frac{1}{s+4}\right) + X(s)\left(\frac{1}{s+3}\right) - 2X(s)\left(\frac{1}{s+4}\right)\left(\frac{1}{s+3}\right)$$

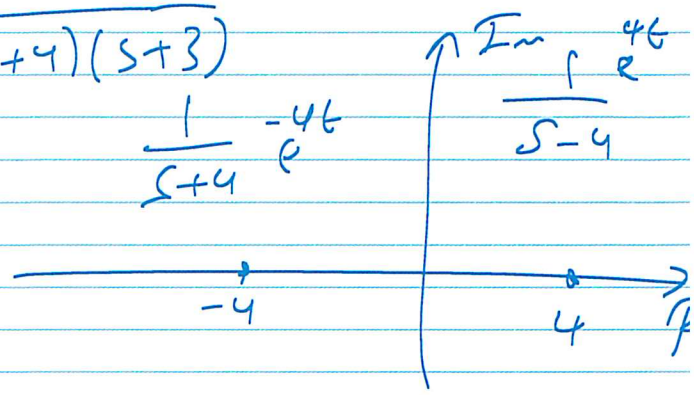
$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{s+4} + \frac{1}{s+3} - 2\left(\frac{1}{s+4}\right)\left(\frac{1}{s+3}\right)$$

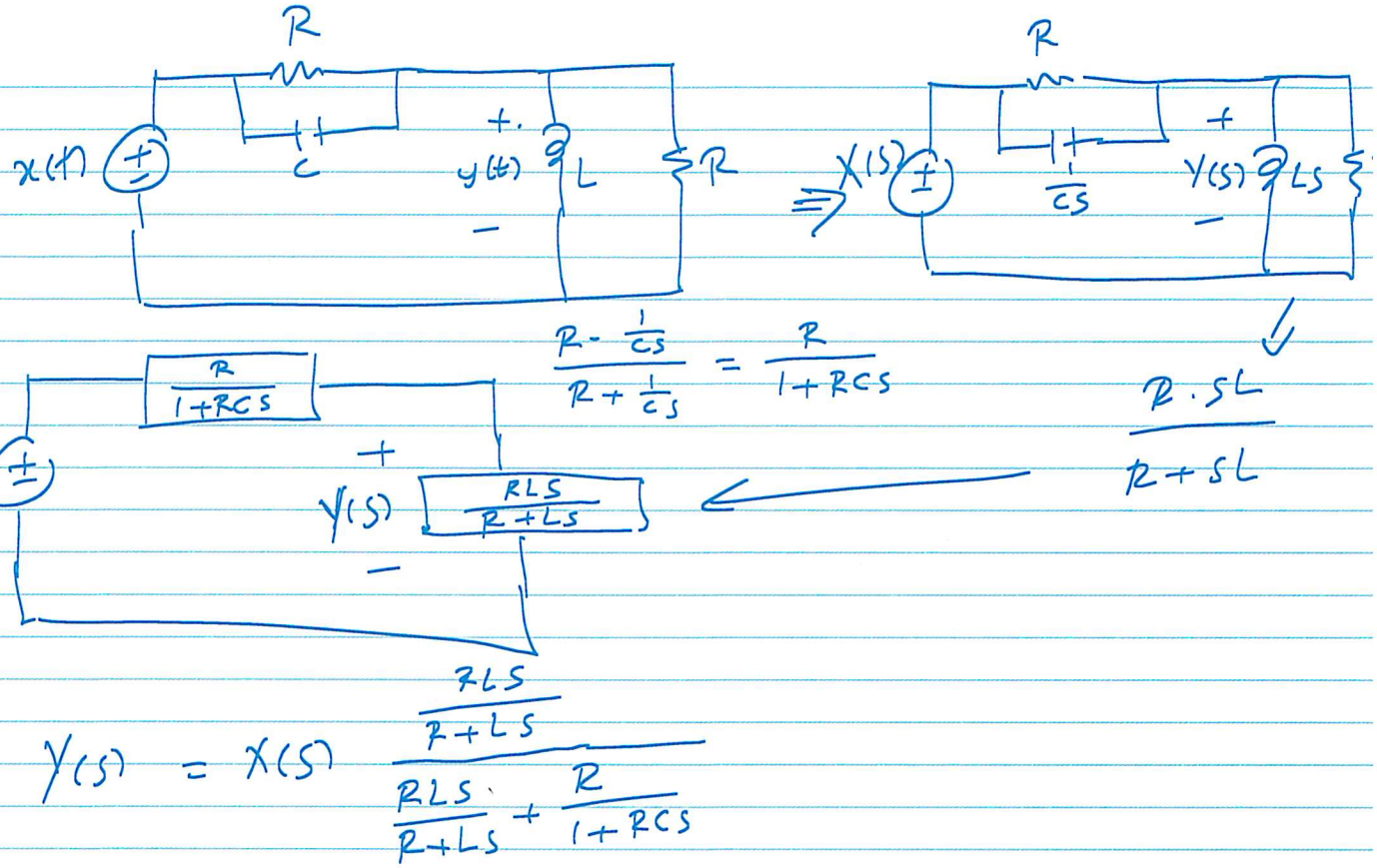
$$H(s) = \frac{1}{s+4} + \frac{1}{s+3} - \frac{2}{(s+4)(s+3)}$$

$\frac{1}{s+4}$ e^{-4t}

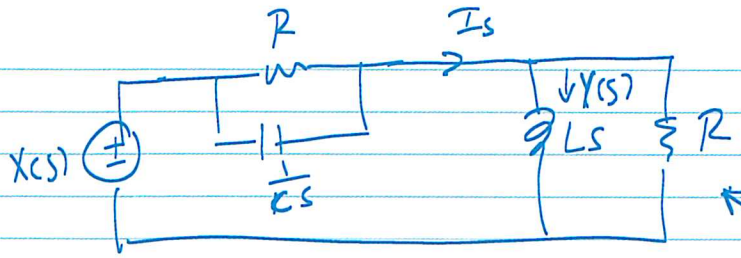
$+4t$

e^{0}



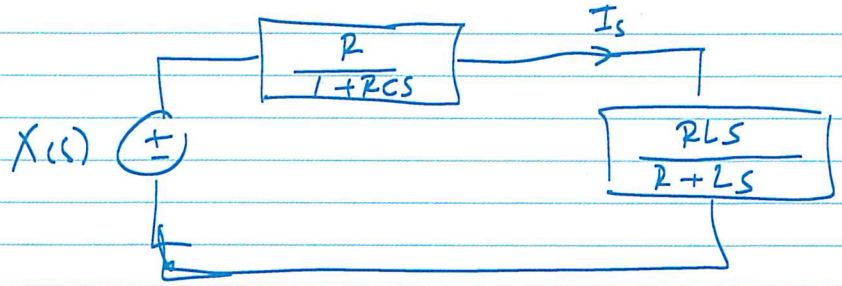


$$Y(s) = X(s) \frac{\frac{RLs}{R+Ls}}{\frac{RLs}{R+Ls} + \frac{R}{1+RCs}}$$



$$Y(s) = I_s\left(\frac{R}{R+Ls}\right)$$

$$= \frac{X(s)}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}\left(\frac{R}{R+Ls}\right.$$



$$\frac{Y(s)}{X(s)} = \frac{\frac{R}{R+Ls}}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}$$

$$I_s = \frac{X(s)}{\frac{R}{1+RCs} + \frac{RLs}{R+Ls}}$$

$$X(s) = \frac{10}{s^2+7s+10}, \text{ find } x(t)$$

$$X(s) = \frac{10}{(s+2)(s+5)} = \frac{C_1}{s+2} + \frac{C_2}{s+5}$$

$$C_1 = (s+2)X(s)\Big|_{s=-2} = \frac{10}{s+5}\Big|_{s=-2} = \frac{10}{3}$$

$$C_2 = (s+5)X(s)\Big|_{s=-5} = \frac{10}{s+2}\Big|_{s=-5} = \frac{10}{-3} = -\frac{10}{3}$$

$$X(s) = \frac{10/3}{s+2} - \frac{5}{s+5}$$

$$x(t) = \frac{10}{3}e^{-2t} - \frac{10}{3}e^{-5t}, \quad t \geq 0$$

$$X(s) = \frac{10}{(s+2)^2(s+5)} = \frac{C_1}{s+2} + \frac{C_2}{(s+2)^2} + \frac{C_3}{s+5}$$

$$C_2 = (s+5)X(s)\Big|_{s=-5} = \frac{10}{(s+2)^2}\Big|_{s=-5} = \frac{10}{(-3)^2} = \frac{10}{9}$$

$$C_2 = (s+2)^2 X(s)\Big|_{s=-2} = \frac{10}{s+5}\Big|_{s=-2} = \frac{10}{3}$$

$$\frac{10}{(s+2)^2(s+5)} = \frac{C_1(s+2)(s+5) + C_2(s+5) + C_3(s+2)^2}{(s+2)^2(s+5)}$$

$$10 = C_1(s^2+7s+10) + C_2 s + C_2 5 + C_3(s^2+4s+4)$$

$$10 = (C_1+C_3)s^2 + (7C_1 + C_2 + 4C_3)s + (10C_1 + 5C_2 + 4C_3)$$

$$C_1 + C_3 = 0 \Rightarrow C_1 = -\frac{10}{9}$$

$$7C_1 + C_2 + 4C_3 = 0 \qquad 10C_1 + 5C_2 + 4C_3 = 10$$

$$C_{r-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}\left[(s-p_1)^r X(s)\right]\right]_{s=p_1}$$

$r = 2$

$$C_1 = \frac{1}{1!}\left[\frac{d}{ds}\left[(s+2)^2 X(s)\right]\right]\Big|_{s=-2}$$

$$C_1 = \frac{d}{ds}\left[\frac{10}{s+5}\right]\Big|_{s=-2}$$

$$= \frac{d}{ds}\left[10(s+5)^{-1}\right]\Big|_{s=-2}$$

$$= -10(s+5)^{-2}\Big|_{s=-2}$$

$$= \frac{-10}{(s+5)^2}\Big|_{s=-2} = \frac{-10}{(-2+5)^2} = \frac{-10}{3^2} = \frac{-10}{9}$$

$$\boxed{C_1 = \frac{-10}{9}}$$