Note Title 2/20/2018

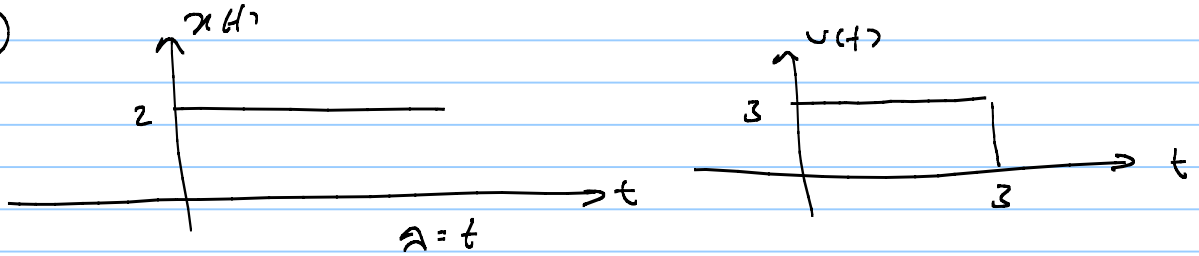
Exam 1 : Lecture 1 — Lecture 5

HW 1 — HW 4

formula sheet will be provided

Solution of Differential Equations won't be in the Exam

(1)



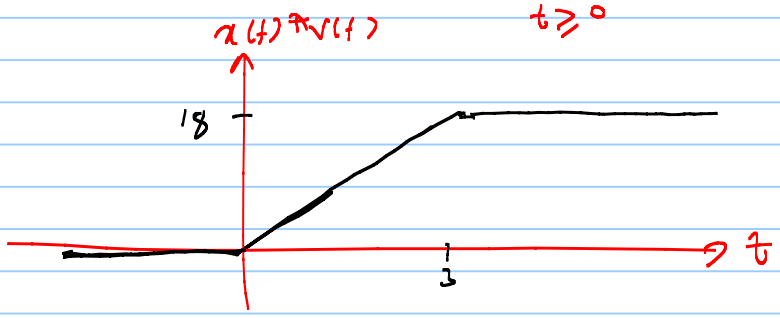
$$x(t) * v(t) = \int_{\lambda=-\infty}^{\lambda=\infty} x(\lambda)\, v(t-\lambda)\, d\lambda$$

$$= \int_{\lambda=0}^{\lambda=t} [2u(\lambda)][3u(t-\lambda) - 3u(t-3-\lambda)]\, d\lambda$$

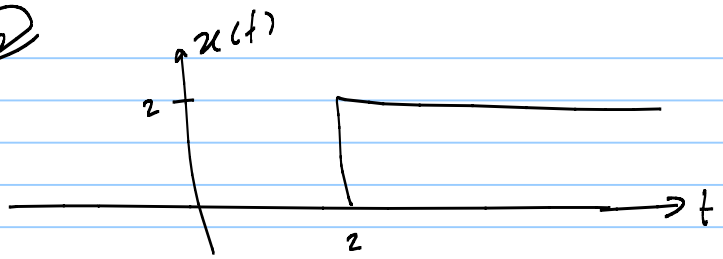
$$= \int_{\lambda=0}^{\lambda=t} [2u(\lambda)][3u(t-\lambda)]\, d\lambda - \int_{\lambda=0}^{\lambda=t-3} [2u(\lambda)][3u(t-3-\lambda)]\, d\lambda$$

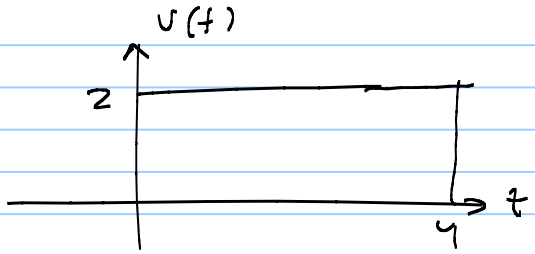
$$= 6\lambda\Big|_0^t - 6\lambda\Big|_0^{t-3}$$

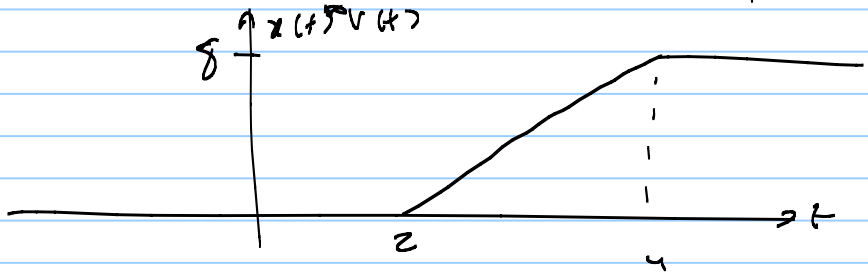
$$= [6t - 0]\Big|_{t\geq 0} - [6t - 18 - 0]\Big|_{t\geq 3}$$



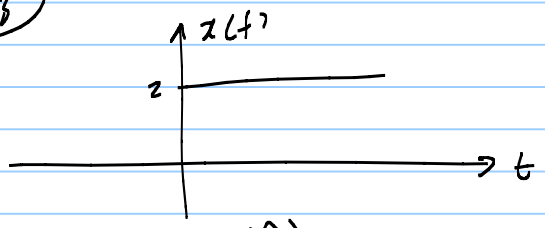
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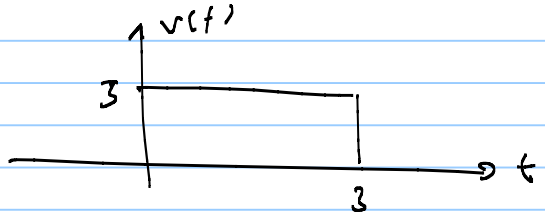


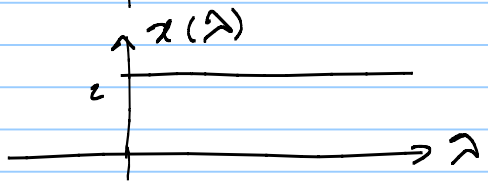


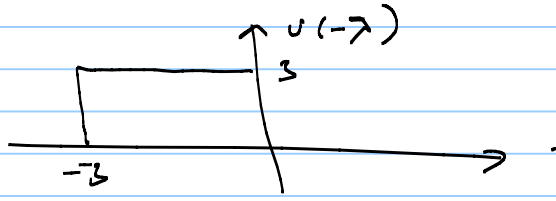


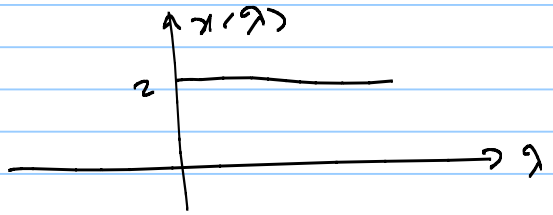
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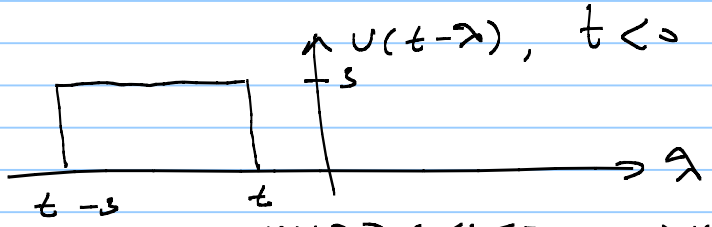


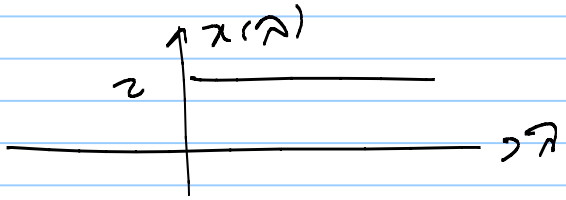


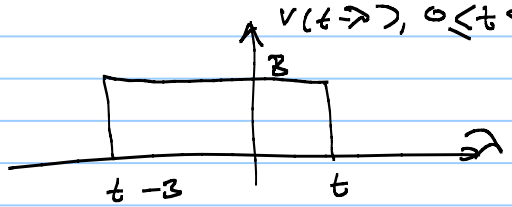


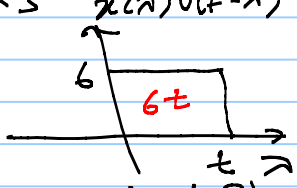


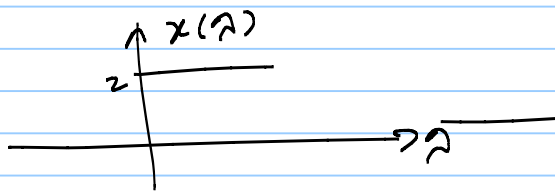


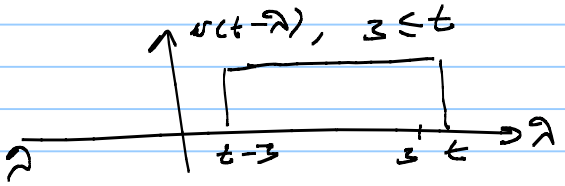


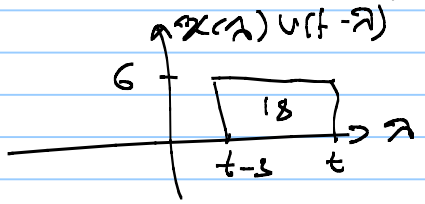








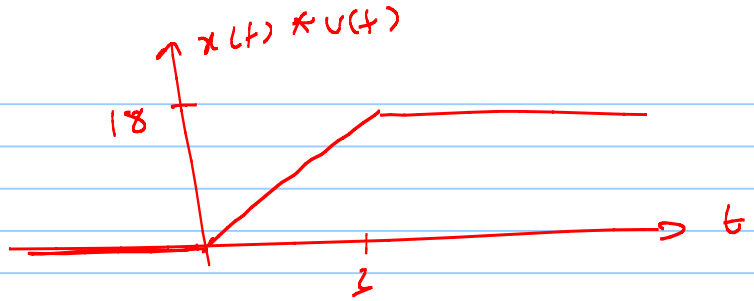




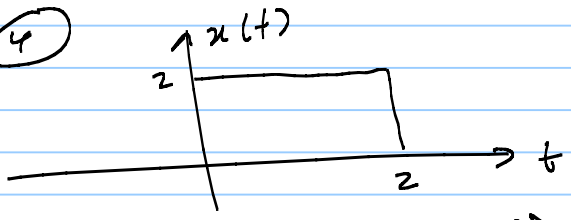
for $t<0$, $x(t)*v(t)=0$

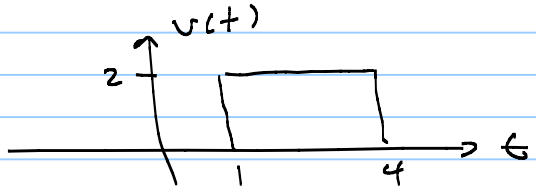
for $0\le t<3$, $x(t)*v(t)=6t$

for $t\ge 3$, $x(t)*v(t)=18$

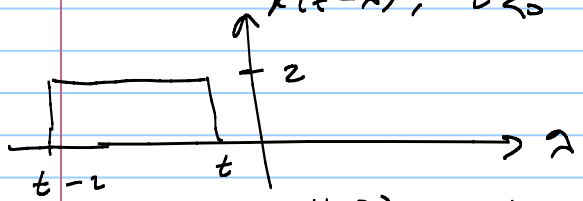


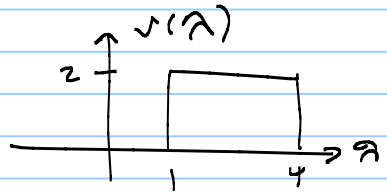
(4)

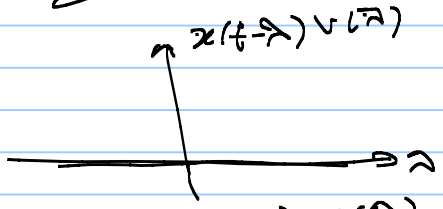


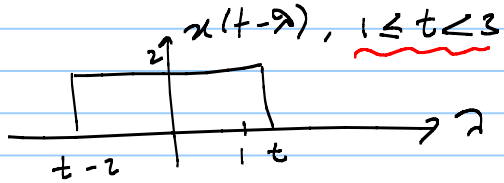


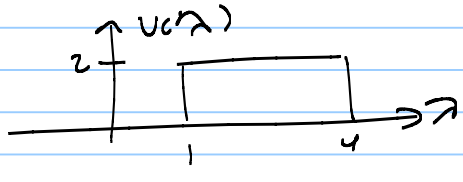
$$x(t) * v(t) = \int_{-\infty}^{\infty} x(\lambda)\, v(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} x(t-\lambda)\, v(\lambda)\, d\lambda$$

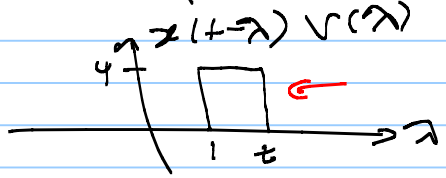


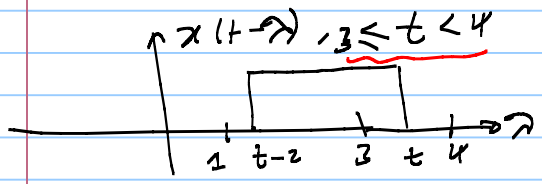


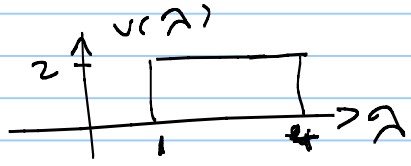


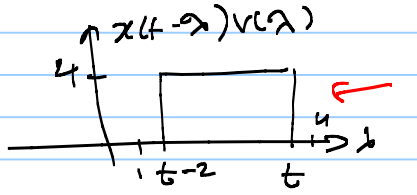


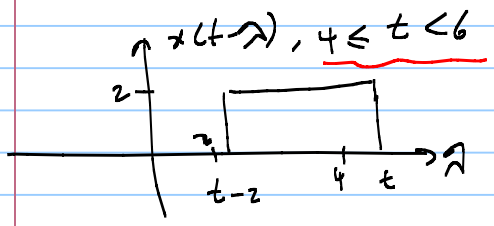


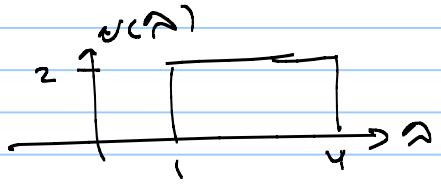


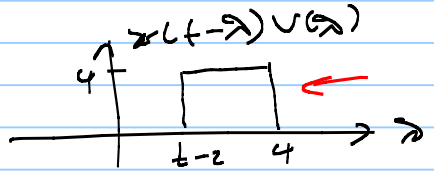










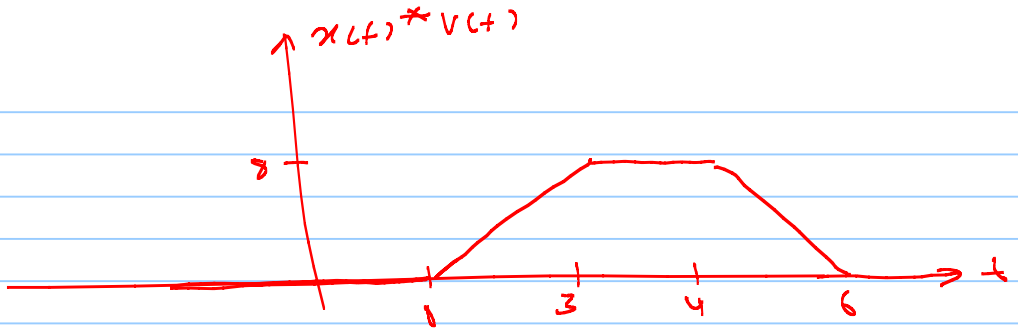


for $t<1$ & $t \geq 6$, $x(t) * v(t) = 0$

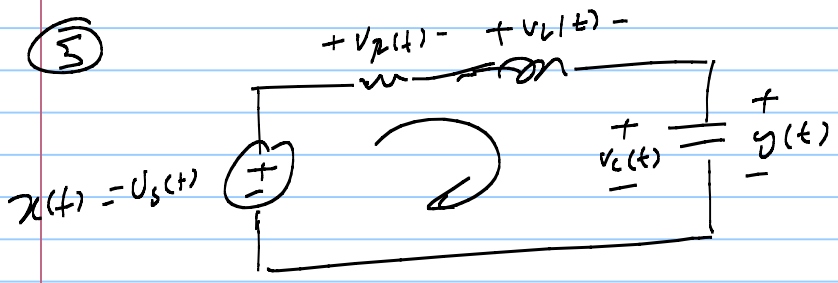
for $1 \leq t < 3$, $x(t) * v(t) = \int_{1}^{t} 4\, d\lambda = 4t - 4$ ←

for $3 \leq t < 4$, $x(t) * v(t) = \int_{t-2}^{t} 4\, d\lambda = 8$

for $4 \leq t < 6$, $x(t) * v(t) = \int_{t-2}^{4} 4\, d\lambda = 4\lambda \Big|_{t-2}^{4} = -4t + 24$



(5)



KVL

$$-x(t) + v_R(t) + v_L(t) + y(t) = 0$$

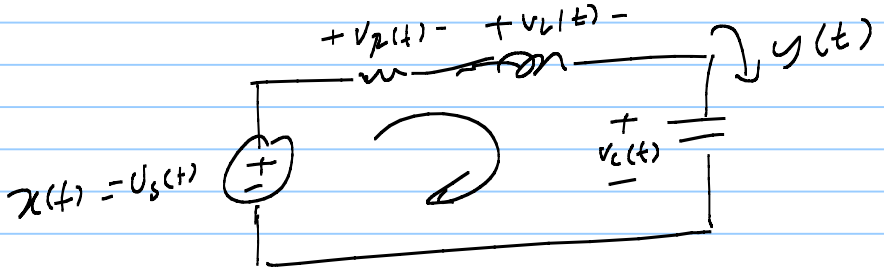
$$-x(t) + Ri(t) + L\frac{di(t)}{dt} + y(t) = 0$$

But $$i(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$$

$$\therefore -x(t) + RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t) = 0$$

$$\frac{d^2y(t)}{dt} + \frac{R}{2}\frac{dy(t)}{dt} + \frac{1}{LC}y(t) = \frac{1}{LC}x(t)$$

(6)



KVL

$$-x(t) + v_R(t) + v_L(t) + v_c(t) = 0$$

$$-x(t) + Ry(t) + L\frac{dy(t)}{dt} + \frac{1}{C}\int y(t)\,dt = 0$$

$$-\frac{dx(t)}{dt} + R\frac{dy(t)}{dt} + L\frac{d^2y(t)}{dt} + \frac{1}{C}y(t) = 0$$

$$\frac{d^2y(t)}{dt} + \frac{R}{2}\frac{dy(t)}{dt} + \frac{1}{2C}y(t) = \frac{1}{2}\frac{dx(t)}{dt}$$

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(7)

$$y[n+1] + 2y[n] = 2x[n]$$

(a) Find the unit-pulse response:

$$h[n+1] + 3h[n] = 2\delta[n]$$

$$h[n+1] = -3h[n] + 2\delta[n] \quad \text{or} \quad h[n] = -3h[n-1] + 2\delta[n-1]$$

$$\therefore\ h[0] = -3h[-1] + 2\delta[-1] = 0$$

$$h[1] = -3h[0] + 2\delta[0] = 2 = 2(-3)^0$$
$$h[2] = -3h[1] + 2\delta[1] = -6 = 2(-3)^1$$
$$h[3] = -3h[2] + 2\delta[2] = 18 = 2(-3)^2$$
$$h[4] = -3h[3] + 2\delta[3] = -54 = 2(-3)^3$$
$$h[5] = -3h[4] + 2\delta[4] = +162 = 2(-3)^4$$
$$h[6] = -486 = 2(-3)^5$$

$$h[n] = 2(-3)^{n-1},\ n \geq 1 \quad \& \quad h[n] = 0,\ n \leq 0$$

(b) Find $y[n]$ if $y[-1] = 0$, $x[n] = 1$, $n \geq 0$

$$y[n+1] = -3y[n] + 2x[n]$$

$$y[n] = -3y[n-1] + 2x[n-1]$$

$$y[0] = -3y[-1] + 2x[-1] = 0$$ ($y[-1]$, $x[-1]$ → 0)

$$y[1] = -3y[0] + 2x[0] = 2$$ ($y[0]$ → 0)

$$y[2] = -3y[1] + 2x[1] = -3(2) + 2$$

$$y[3] = -3y[2] + 2x[2] = 2(-3)^2 + 2(-3) + 2$$

$$y[4] = -3y[3] + 2x[3]$$

$$= 2(-3)^3 + 2(-3)^2 + 2(-3) + 2$$

$$y[n] = \sum_{i=1}^{n} 2(-3)^{n-i}$$

$$y[4] = 2(-3)^3 + 2(-3)^2 + 2(-3)^1 + 2(-3)^0$$

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(7) (a) $y(t) = 2x^2(t)$

$x_1(t) \longrightarrow y_1(t) = 2x_1^2(t)$

$x_2(t) \longrightarrow y_2(t) = 2x_2^2(t)$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = 2\left(x_1(t) + x_2(t)\right)^2$

$y_1(t) + y_2(t) \neq y_3(t) \Longrightarrow$ non-linear

(b) $y(t) = 2x(t) + 3$

$x_1(t) \longrightarrow y_1(t) = 2x_1(t) + 3$

$x_2(t) \longrightarrow y_2(t) = 2x_2(t) + 3$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = 2\left(x_1(t) + x_2(t)\right) + 3$

$y_1(t) + y_2(t) \neq y_3(t) \Rightarrow$ non-linear

(c)

$$y(t) = \sin(t)\, x(t)$$

$x_1(t) \rightarrow y_1(t) = \sin(t)\, x_1(t)$

$x_2(t) \rightarrow y_2(t) = \sin(t)\, x_2(t)$

$x_1(t) + x_2(t) \rightarrow y_3(t) = \sin(t)\,(x_1(t) + x_2(t))$

$y_1(t) + y_2(t) = y_3(t)$ ✓ linear

But Time varying system

(d)

$$y(t) = \sin(x(t))$$

$x_1(t) \rightarrow y_1(t) = \sin(x_1(t))$

$x_2(t) \rightarrow y_2(t) = \sin(x_2(t))$

$x_1(t) + x_2(t) \rightarrow y_3(t) = \sin(x_1(t) + x_2(t))$

$y_1(t) + y_2(t) \neq y_3(t) \Rightarrow$ non-linear

Time-Invariant system