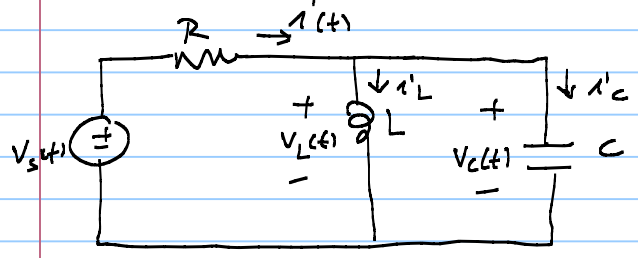
Note Title 9/19/2017

Ex 1: Find the differential equation of the following system. Let

$x(t) = v_s(t)$ and $y(t) = v_c(t)$



$-v_s(t) + R\,i'(t) + v_c(t) = 0$ and $i'(t) = i'_L(t) + i'_C(t)$

$i'_C(t) = C\dfrac{dv_c(t)}{dt}$

$\therefore -v_s(t) + R\,i'_L(t) + RC\dfrac{dv_c(t)}{dt} + v_c(t) = 0$

OR $-x(t) + R\,i'_L(t) + RC\dfrac{dy(t)}{dt} + y(t) = 0$

$v_L(t) = L\dfrac{di'_L(t)}{dt}$, But $v_L(t) = v_c(t) \Rightarrow v_c(t) = L\dfrac{di'_L(t)}{dt}$

$\therefore i'_L(t) = \dfrac{1}{L}\int_{-\infty}^{t} v_c(t)\,dt = \dfrac{1}{L}\int_{-\infty}^{t} y(t)\,dt$

$\therefore -x(t) + \dfrac{R}{L}\int_{-\infty}^{t} y(t)\,dt + RC\dfrac{dy(t)}{dt} + y(t) = 0$

OR $-\dfrac{dx(t)}{dt} + \dfrac{R}{L}y(t) + RC\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} = 0$

$\therefore \dfrac{d^2y(t)}{dt^2} + \dfrac{1}{RC}\dfrac{dy(t)}{dt} + \dfrac{1}{LC}y(t) = \dfrac{1}{RC}\dfrac{dx(t)}{dt}$